



A TIME-MULTISCALE MODEL ORDER REDUCTION METHOD IN NONLINEAR SOLID MECHANICS

P. Ladevèze , D. Néron , S. Rodriguez and R. Scanff
LMT (ENS Paris-Saclay / CNRS / Université Paris-Saclay)

coll G. Nahas , P.E Charbonnel
CEA , IRSN

coll U.Nackenhorst, A. Fau, M. Bhattacharyya, S.Alameddin ,
Hanover University



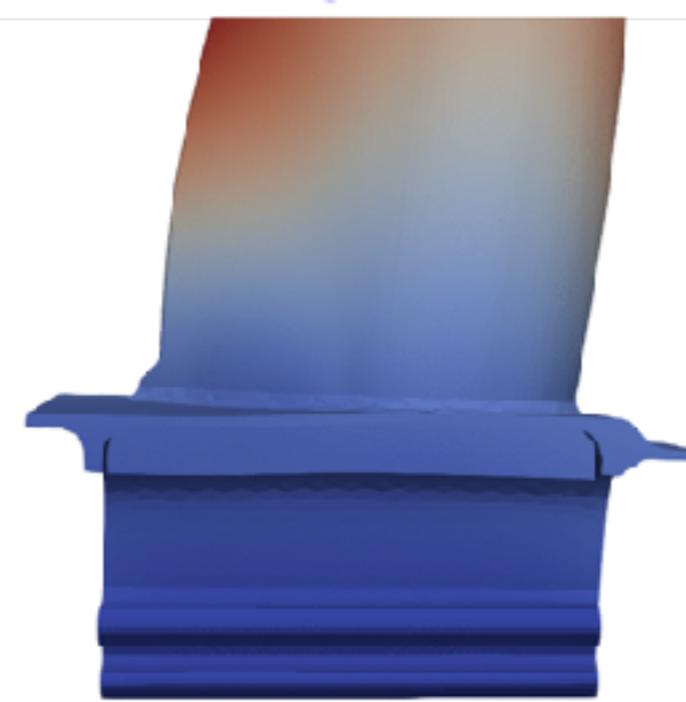
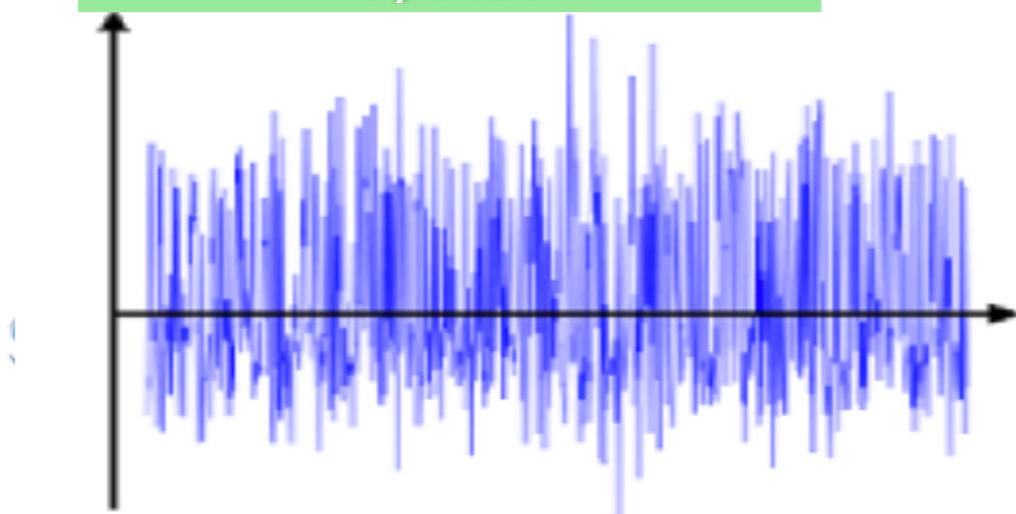
école
normale
supérieure
paris-saclay



université
PARIS-SACLAY

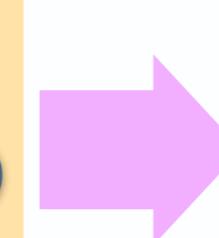
Motivation

fatigue loadings with
large number of
cycles

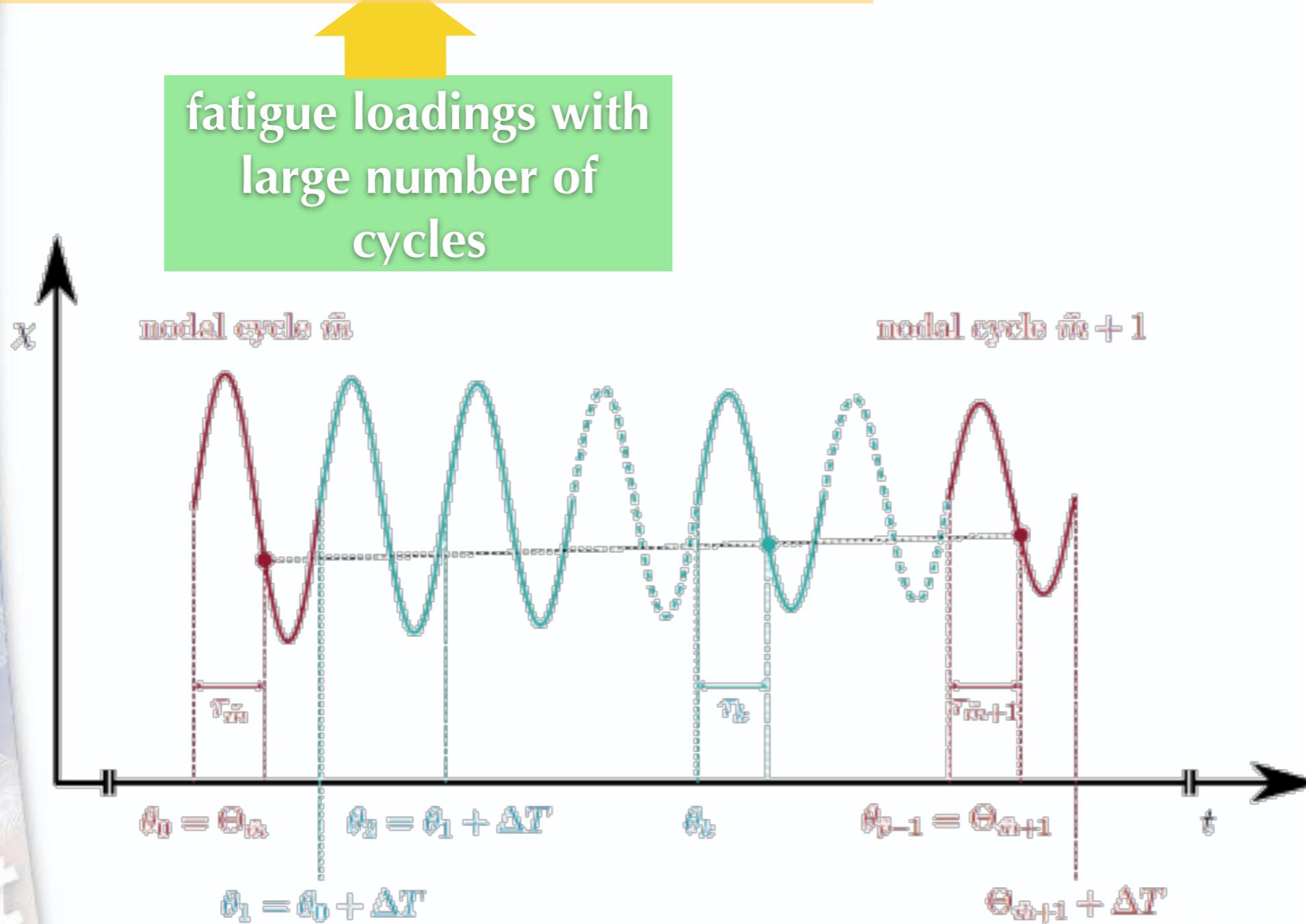


Motivation

To compute time-dependent nonlinear problems (viscoplasticity + fatigue damage)

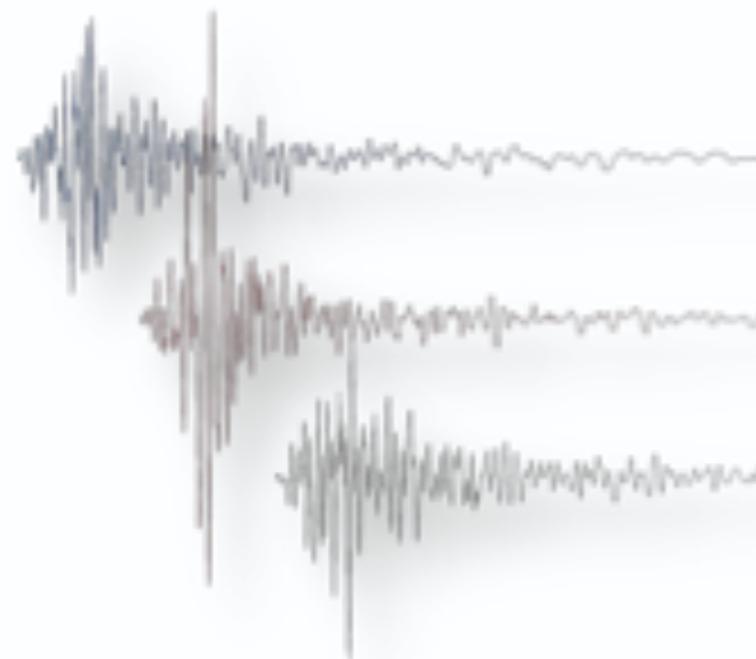


initiation of a macrocrack



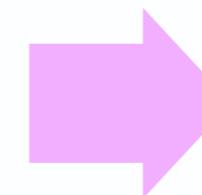
Motivation

seismic loadings problems



Motivation

To compute time-dependent nonlinear problems (viscoplasticity + damage)

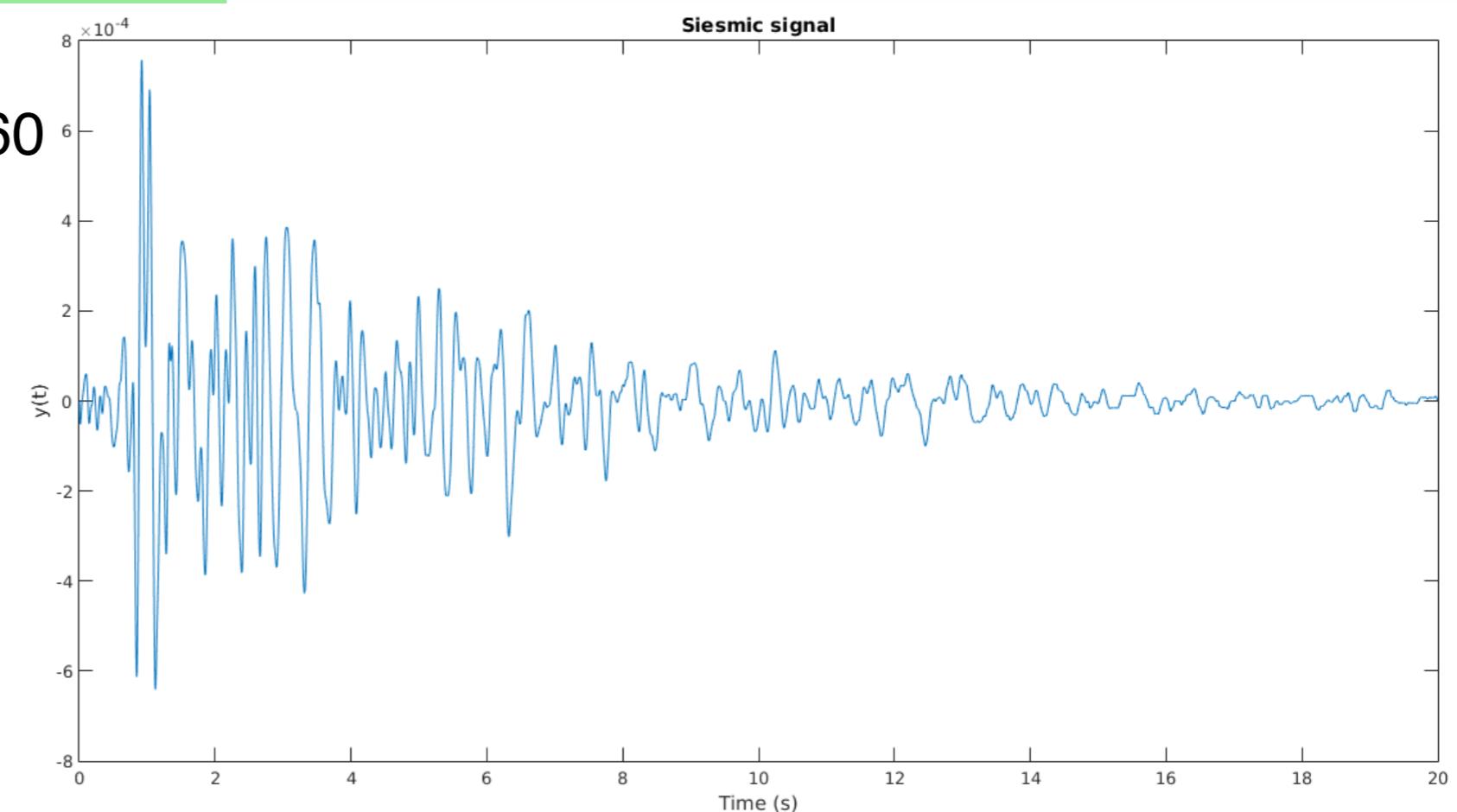


damage evaluation

seismic loadings

Worst earthquake : Chili 1960

- Richter scale: 9.5
- duration : 10mn



Motivation

Our answer : two levels of complexity reduction

- ◆ A signal theory with two time scales
- ◆ The time-multiscale PGD



Another motivation

■ **MOR methods : intrusive → engineering diffusion** ↗↗↗

■ **LATIN-PGD : version non intrusive**

- Very general: +
- Performance: —
- Applications (in progress) in SAMCEF (coll SIEMENS)
and in CASTEM (coll CEA)

Outline

1. A signal theory :a ROM for the loading

2. A new PGD approach : multiscale in time and non intrusive

3. First illustrations

4. Conclusion-Prospects

Outline

1. A signal theory :a ROM for the loading

2. A new PGD approach : multiscale in time and non intrusive

3. First illustrations

4. Conclusion-Prospects

A signal theory: a ROM-loading

Specific aim: minimum of time shape functions

(wavelet theory:
too much terms)

■ Two time-scale approximation (micro : harmonic funct.)

$$s_d(t) : \sum_{i=0}^m \left(\varphi_i^R(t) \cos 2\pi \frac{\tau}{\underline{\tau}_i} + \varphi_i^I(t) \sin 2\pi \frac{\tau}{\underline{\tau}_i} \right) \Big|_{\tau=t} \quad \underline{\tau}_0 = \infty$$

($\underline{\tau}_i$ -mode(H))

- t : « macro » time
- τ : « micro » time

$\underline{\tau}_i \leq \frac{1}{N}$ macro-time scale

($N = 3$)

Characteristics: $(\underline{\tau}_i, \underbrace{\varphi_i^R, \varphi_i^I}_{\text{macro functions}} \mid i \in 0, 1, \dots, m)$

A signal theory: a ROM-loading

■ Two time-scale approximation (micro : harmonic funct.)

$$s_d(t) : \sum_{i=0}^m \underline{n}(\tau, \underline{\tau}_i)^T \mathbf{S}(t) \underline{A}_{|\tau=t}^i$$

$$\underline{n}(\tau, \underline{\tau}_i) : \begin{bmatrix} \cos 2\pi \frac{\tau}{\underline{\tau}_i} \\ \frac{\underline{\tau}_i}{\tau} \\ \sin 2\pi \frac{\tau}{\underline{\tau}_i} \end{bmatrix}$$

$$\mathbf{S}(t) = [\psi_1, \dots, \psi_m](t) \quad \underline{A} = \begin{bmatrix} a_1^R & a_1^I \\ \vdots & \vdots \\ a_m^R & a_m^I \end{bmatrix}$$

FE basis

A signal theory: a ROM-loading

■ Best approximation (micro : harmonic funct.)

$$s_m(t) : (\tau_i, \underline{A}^i \quad | \quad i \in 0, 1, \dots, m) \rightsquigarrow \mathcal{S}_m^{(0, T)}$$

$$s_m(t) = \arg \min_{s'_m \in \mathcal{S}_m^{(0, T)}} \frac{1}{T} \int_0^T (s_d(t) - s'_m(t))^2 dt$$

A signal theory: a ROM-loading

■ Computational method: « greedy » (micro : harmonic funct.)

New correction: $(s_{m+1} - s_m) : (\underline{\tau}, \underline{A}) \rightsquigarrow \underline{n}(\underline{\tau}, \underline{\tau})^T S \underline{A}$

$$\frac{1}{\underline{\tau}} = \arg \max_{\tau' \geq 0} \underline{R}^T \mathbb{M}^{-1} \underline{R}$$
$$\underline{A} = \mathbb{M}^{-1} \underline{R}_{|\underline{\tau}}$$

$$\underline{R}_j = \text{FT}[(s_d - s_m)\psi_j] \times \frac{1}{T}$$

$$\mathbb{M}_{ij} \approx \frac{1}{T} \int_0^T \psi_i \psi_j dt$$

Error : 1/ (6,6xN²) for P1
1/ (165xN³) for P2 (N = 3)

+ strict separation of computed periods

A signal theory: a ROM-loading

■ Computational method: « greedy » (micro : harmonic funct.)

Convergence proof : data = finite sum of modes (H)

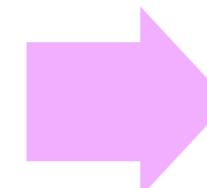
A signal theory: a ROM-loading

■ Other writing

$$s_m(t) = \sum_{j=1}^q \underbrace{\widetilde{\psi_j(t)}}_{\text{FE shape function}} \times \underbrace{\left\{ \sum_{i=0}^m \theta_j^i(\tau, \underline{\tau}_i) \right\}}_{\substack{j-\text{nodal contribution} \\ (\text{harmonic functions})}} |_{\tau=t}$$

■ Theory extension

harmonic function
sum of \underline{T}_k -modes(H)



arbitrary periodic function
sum of \underline{T}_k -modes(P)

A signal theory: a ROM-loading

Theory extension (micro: arbitrary periodic funct.)

\underline{h} : mother periodic function

$$\underline{\tau}_k - \text{mode}(P) = \sum_{j=1}^q \Psi_j(t) \left[a_R^j \underline{h}_R(\tau, \underline{\tau}_k) + a_J^j \underline{h}_J(\tau, \underline{\tau}_k) \right]$$

$$\begin{cases} \underline{h}_R & \text{even} \\ \underline{h}_J & \text{odd} \end{cases}$$

$$\sum_{-\underline{\tau}_k/2}^{\underline{\tau}_k/2} (\underline{h}_R)^2 d\tau = \sum_{-\underline{\tau}_k/2}^{\underline{\tau}_k/2} (\underline{h}_J)^2 d\tau = \underline{\tau}_k/2$$

A signal theory: a ROM-loading

■ Theory extension (micro: arbitrary periodic funct.)

$$\underline{\tau} - \text{mode (P)} = \underline{n}(\tau, \underline{\tau})^T \underline{S_A}$$

$$\underline{n} : \begin{matrix} h_R \\ h_I \end{matrix}$$

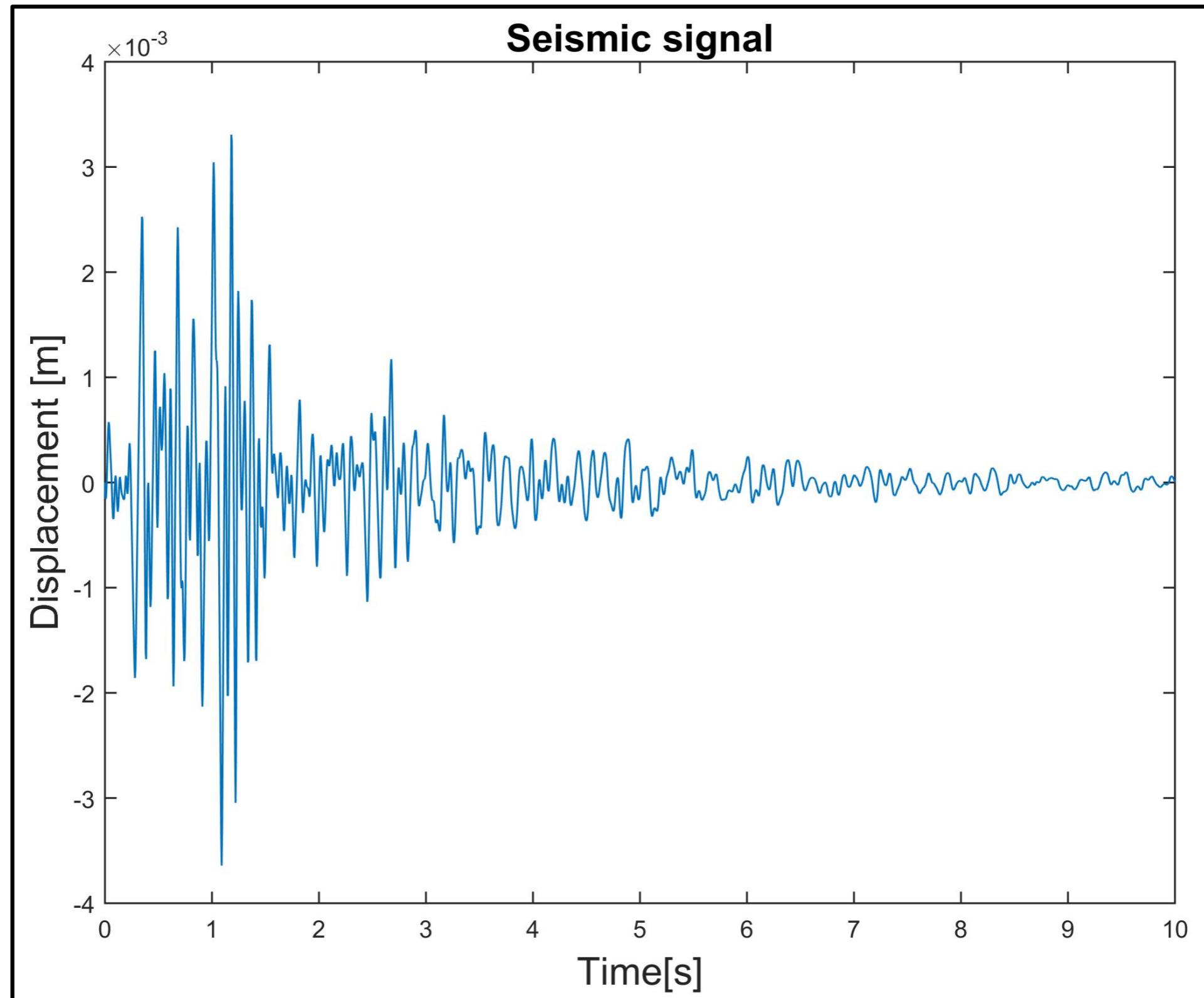
$$\underline{\tau} \leq \frac{1}{N} \text{macro-time scale}$$

(N = 3)



A signal theory: a ROM-loading Illustration I

■ Data



A signal theory: a ROM-loading Illustration I

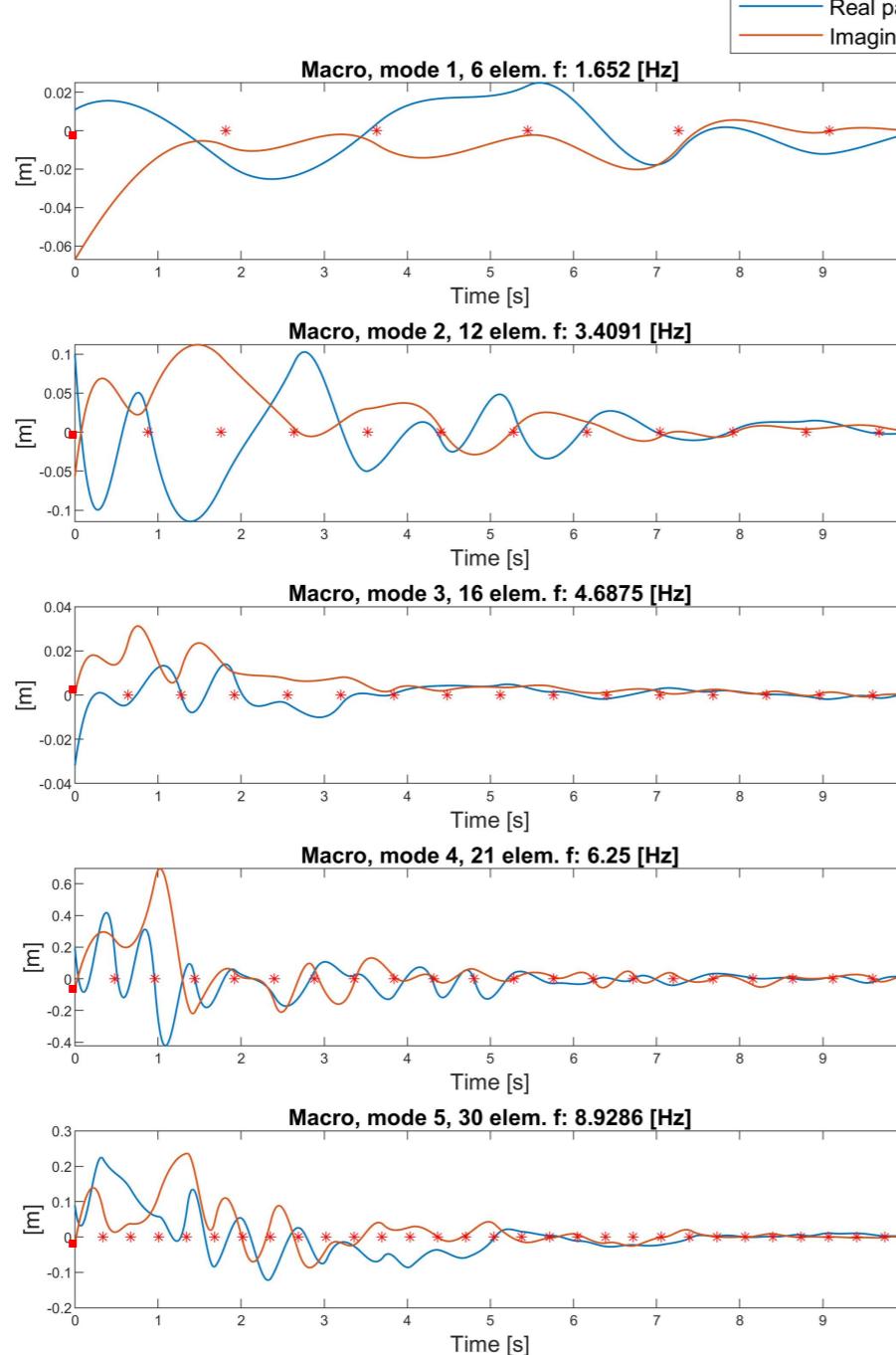
■ ROM- loading with P2 macro-elements

number of components : 8 modes(H)

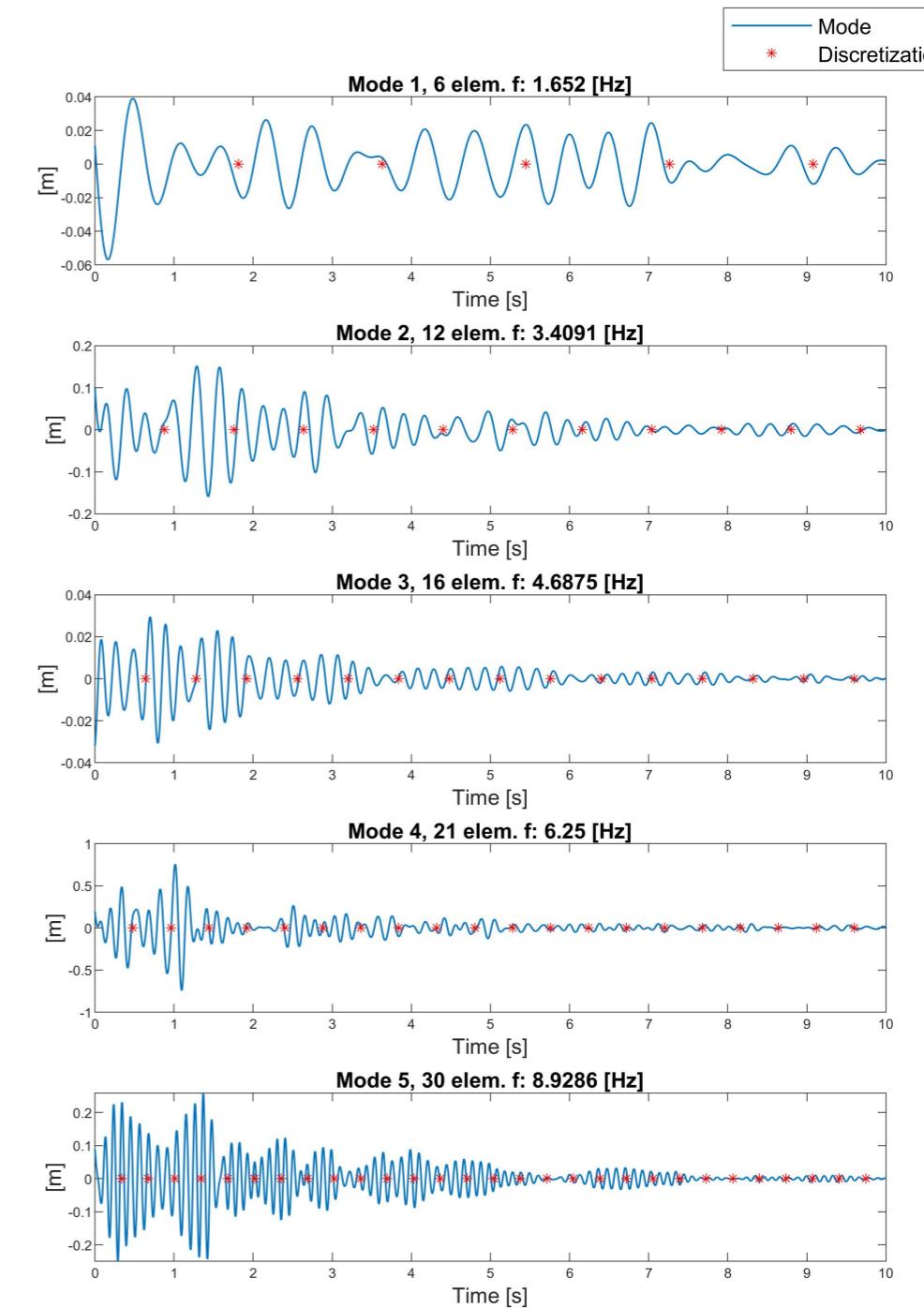
accuracy : $5 \cdot 10^{-2}$

A signal theory: a ROM-loading Illustration I

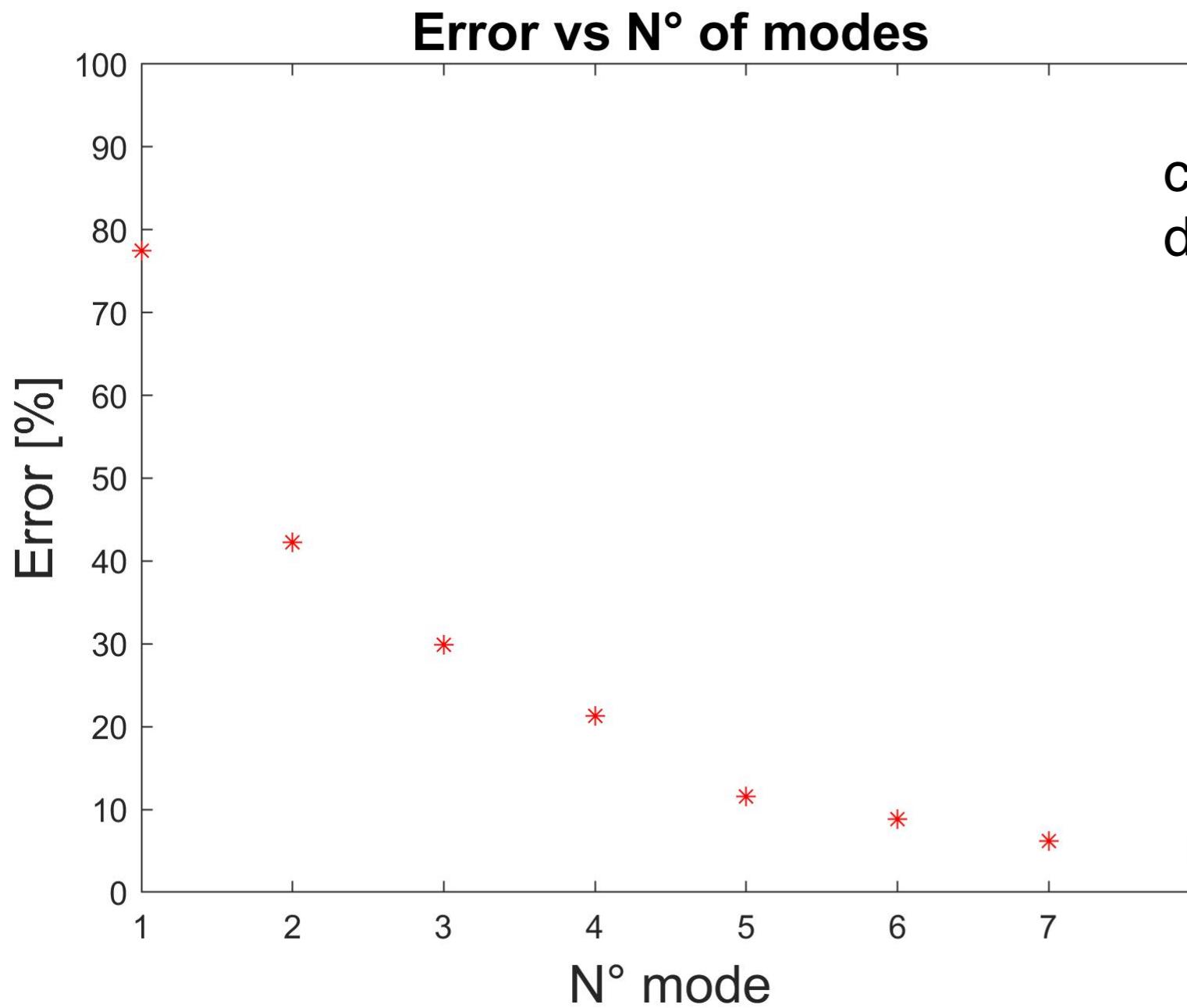
Macro functions



Modes

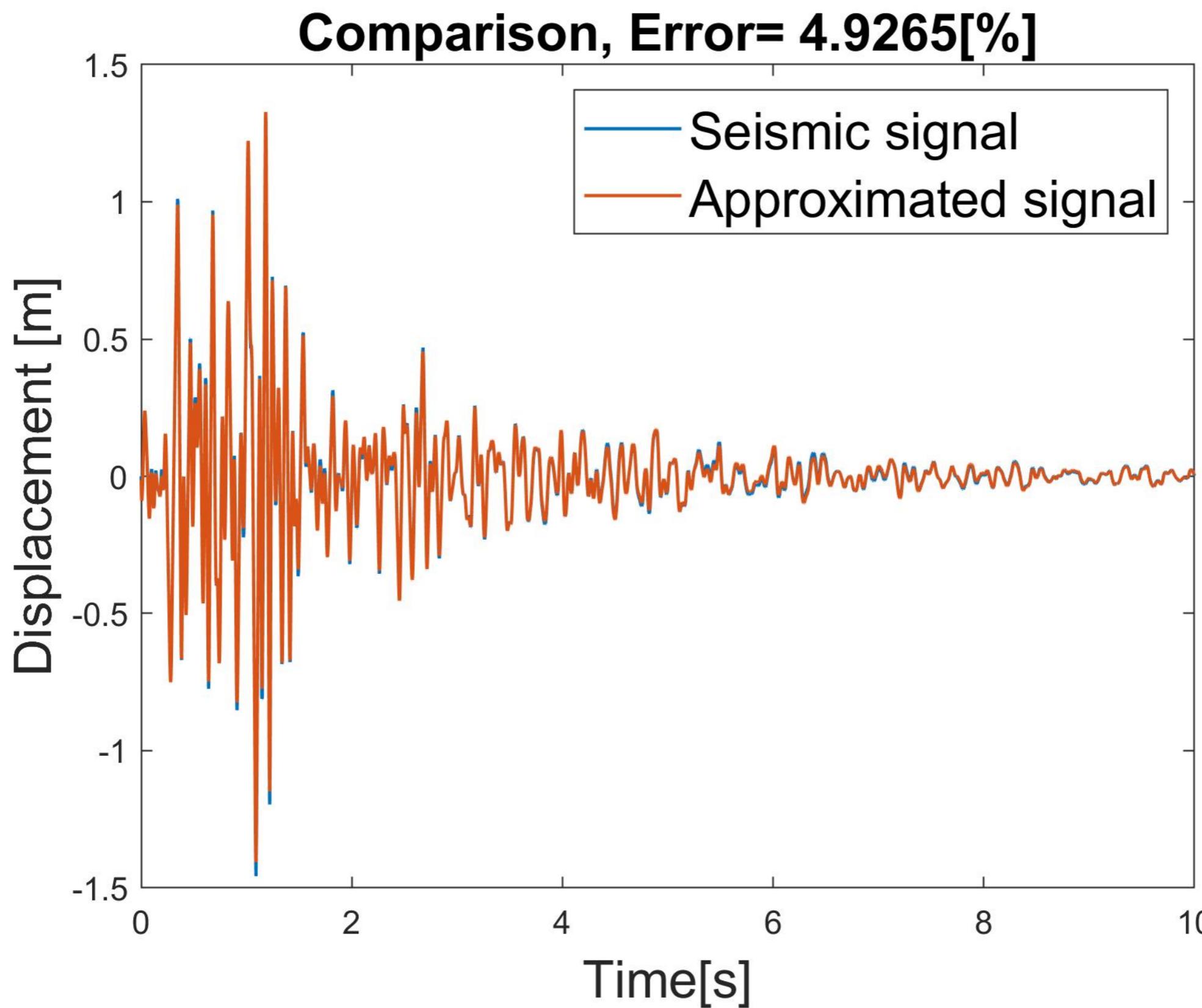


A signal theory: a ROM-loading Illustration I

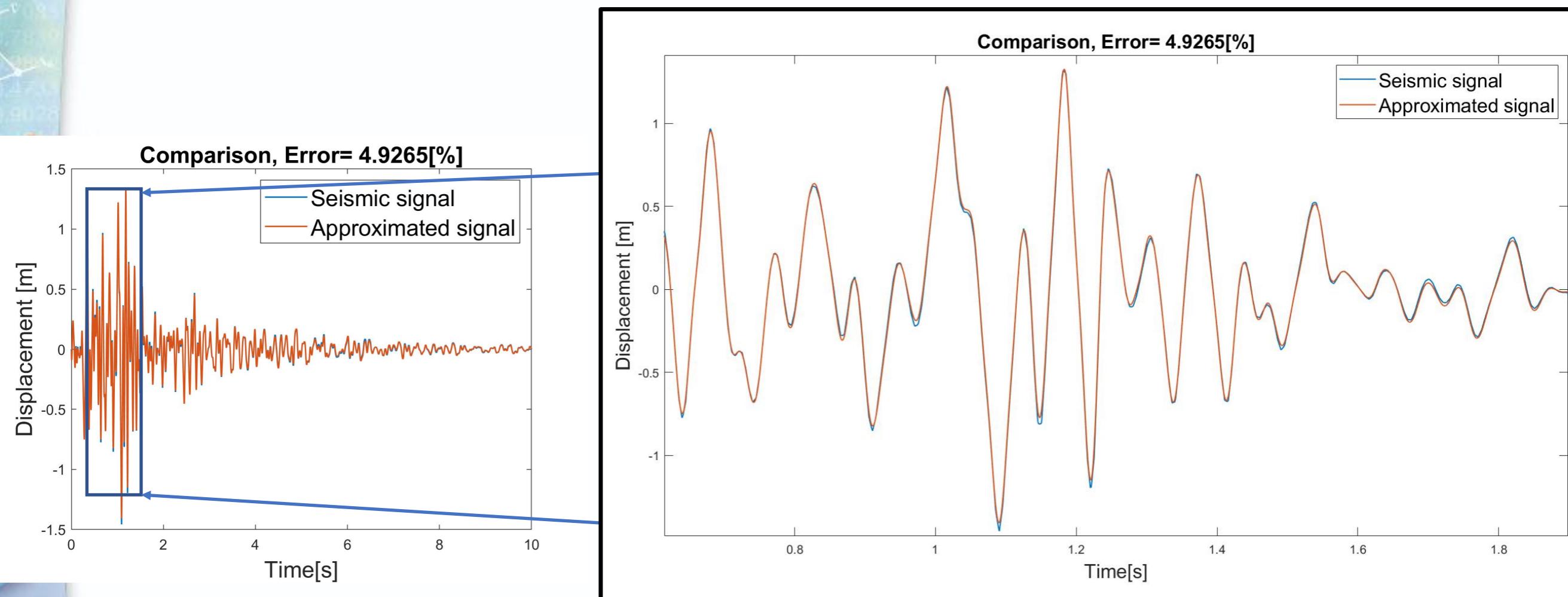


classical time discretisation : 15 000
dofs for 8 modes : 690

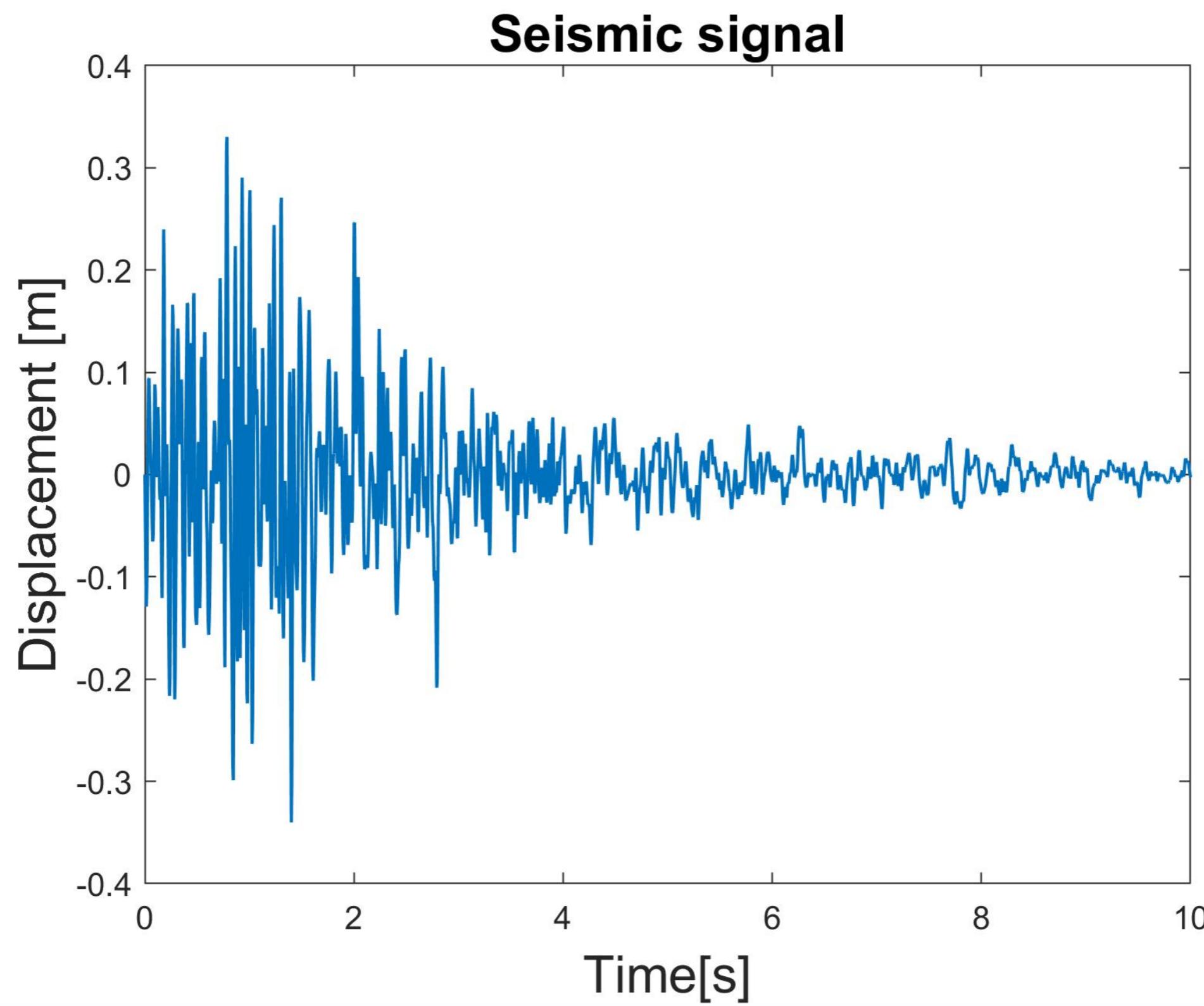
A signal theory: a ROM-loading Illustration I



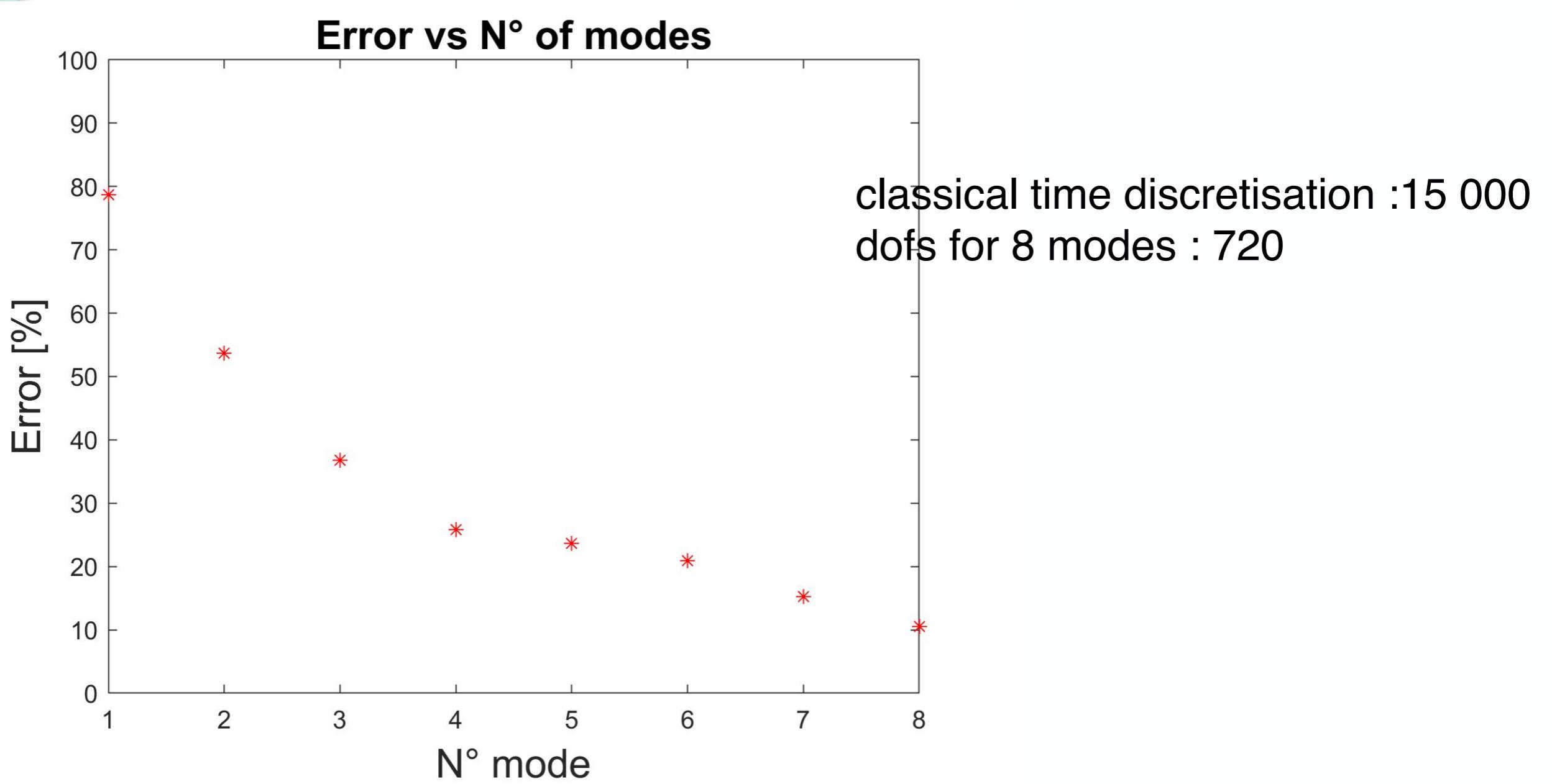
A signal theory: a ROM-loading Illustration I



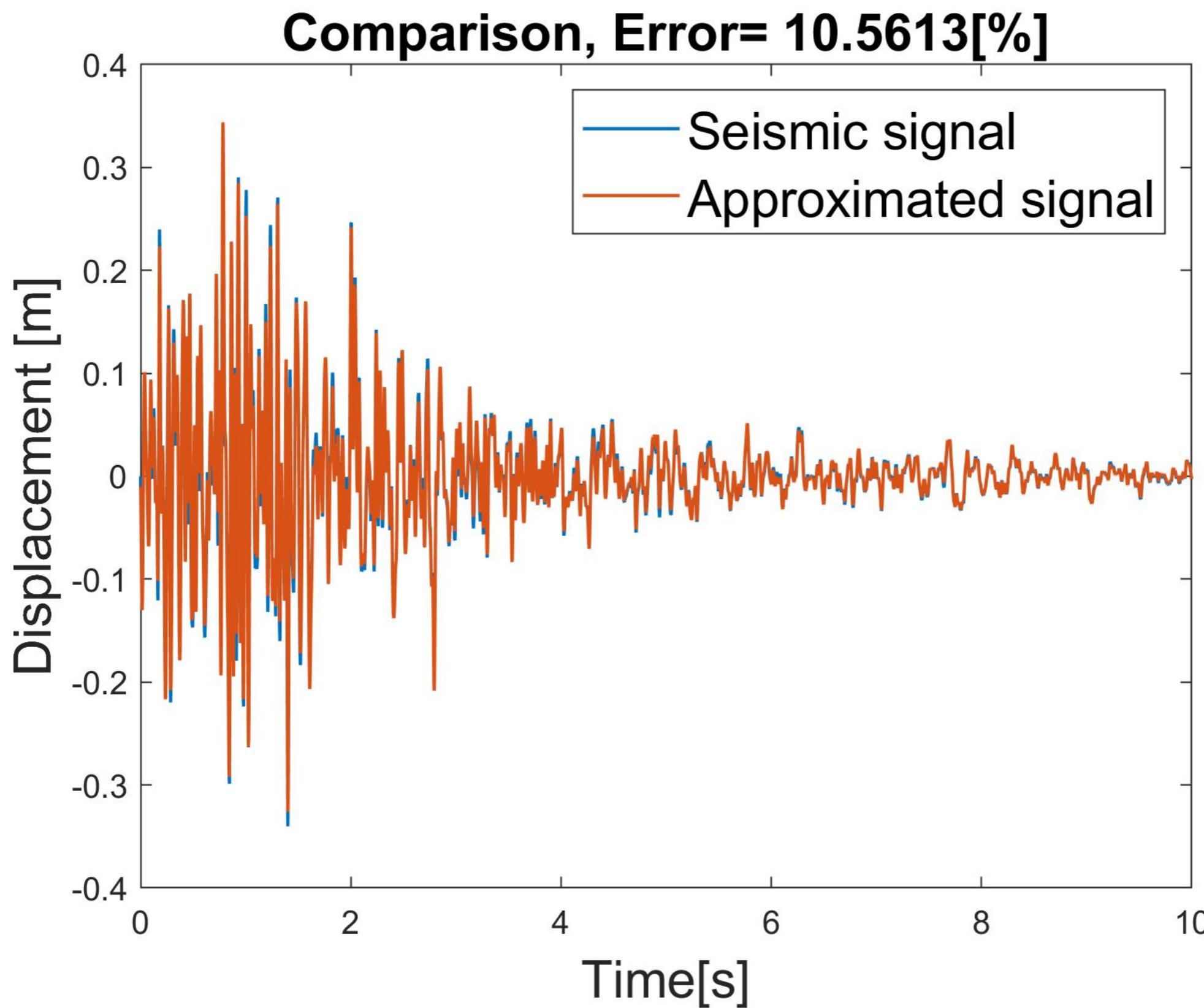
A signal theory: a ROM-loading Illustration 2



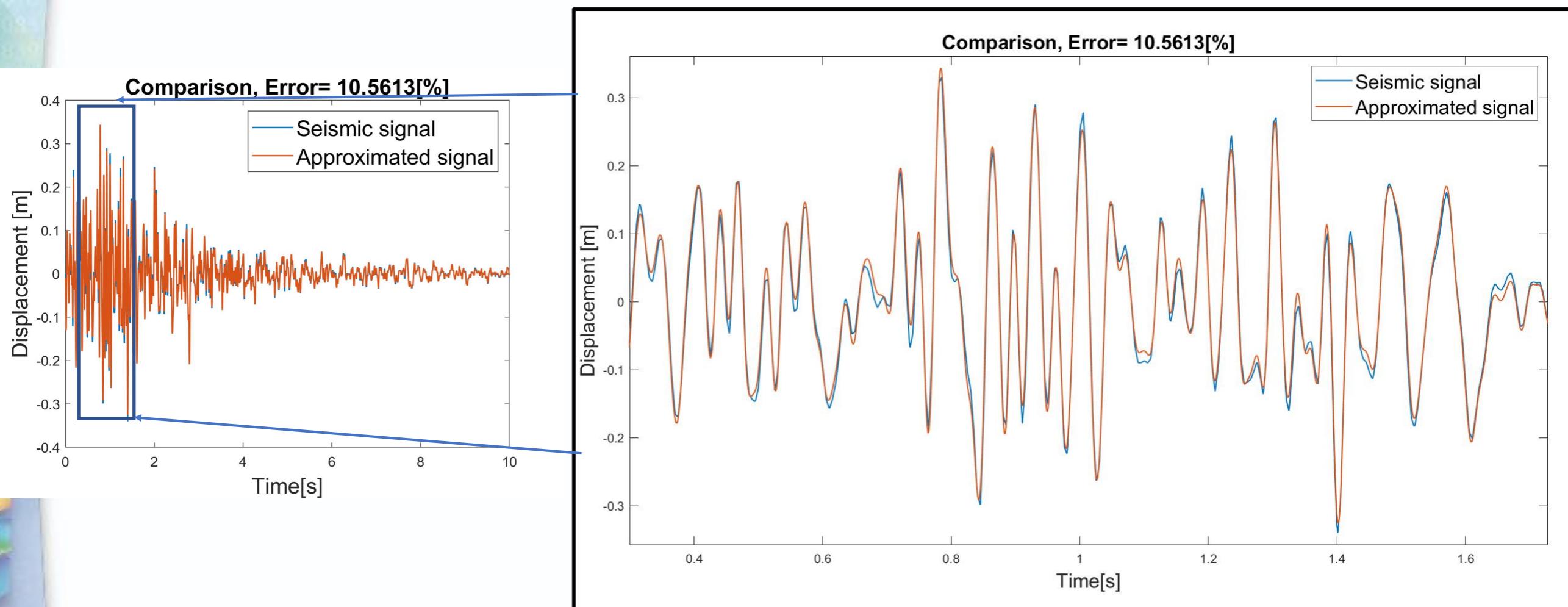
A signal theory: a ROM-loading Illustration 2



A signal theory: a ROM-loading Illustration 2



A signal theory: a ROM-loading Illustration 2



Outline

1. A signal theory :a ROM for the loading

2. A new PGD approach : multiscale in time and non intrusive

3. First illustrations

4. Conclusion-Prospects

Reference discretized problem to be solved over $[0, T] \times \Omega$

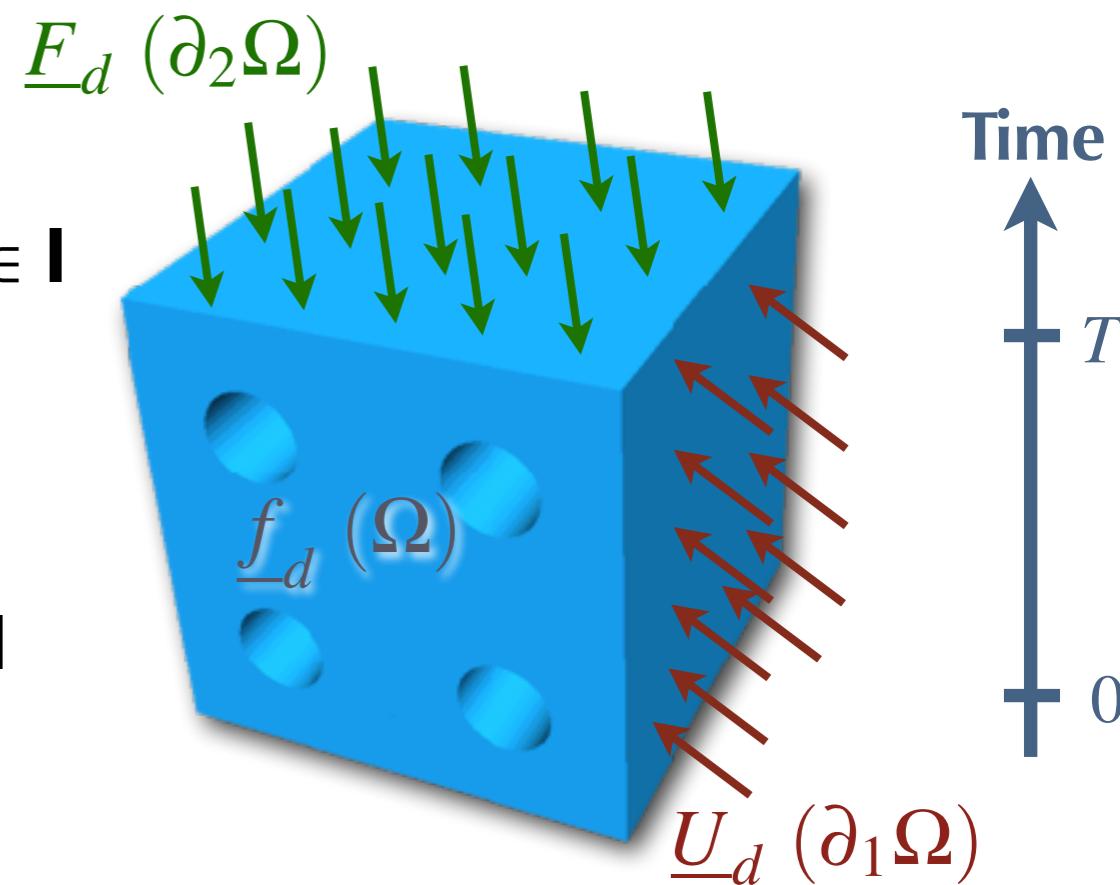
FE-framework

■ Discretized time-space domain $[0, T] \times \Omega$ + fields-approximation

- nodal points : $x_j \quad j \in 1 \dots n$
- time discretization : $t_i \quad i \in 1 \dots m \quad \text{or} \quad t_i \in I$

■ Fundamental quantities

- nodal « displacement » vector : $\underline{u}(t) \quad t \in I$
- nodal « force » vector : $\underline{F}(t) \quad t \in I$



Work : $\underline{F}(t)^T \underline{u}(t) \quad t \in I$

Reference discretized problem to be solved over $[0, T] \times \Omega$

Formulation FE-software

Find $(\underline{u}(t), \underline{F}(t))$ $t \in I$ such that:

■ Equilibrium equation

A_d

$$\forall t \in I \quad \underline{F}(t) = \underline{F}_d(t, \underline{u}(t)) \text{ (given)}$$

■ Constitutive relation

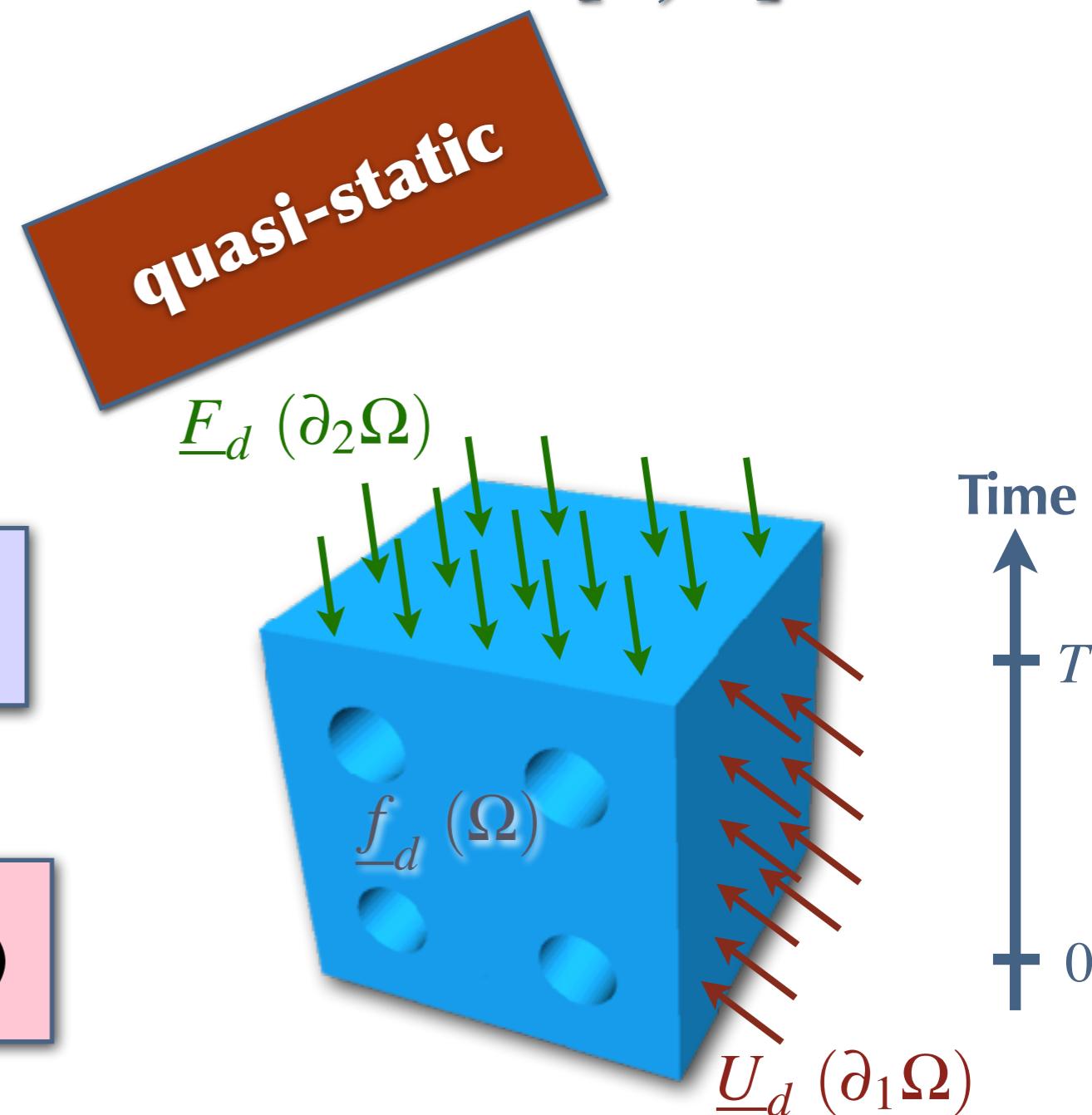
Γ

$$\forall t \in I \quad \underline{F}(t) = \hat{\mathbf{H}}(t, \underline{u}(\tau) \mid \tau \leq t)$$

with

$$\hat{\mathbf{H}}(t, \underline{u}(\tau) \mid \tau \leq t) = \sum_{e \in E} \hat{\mathbf{H}}_e(t, \underline{u}_e(\tau) \mid \tau \leq t)$$

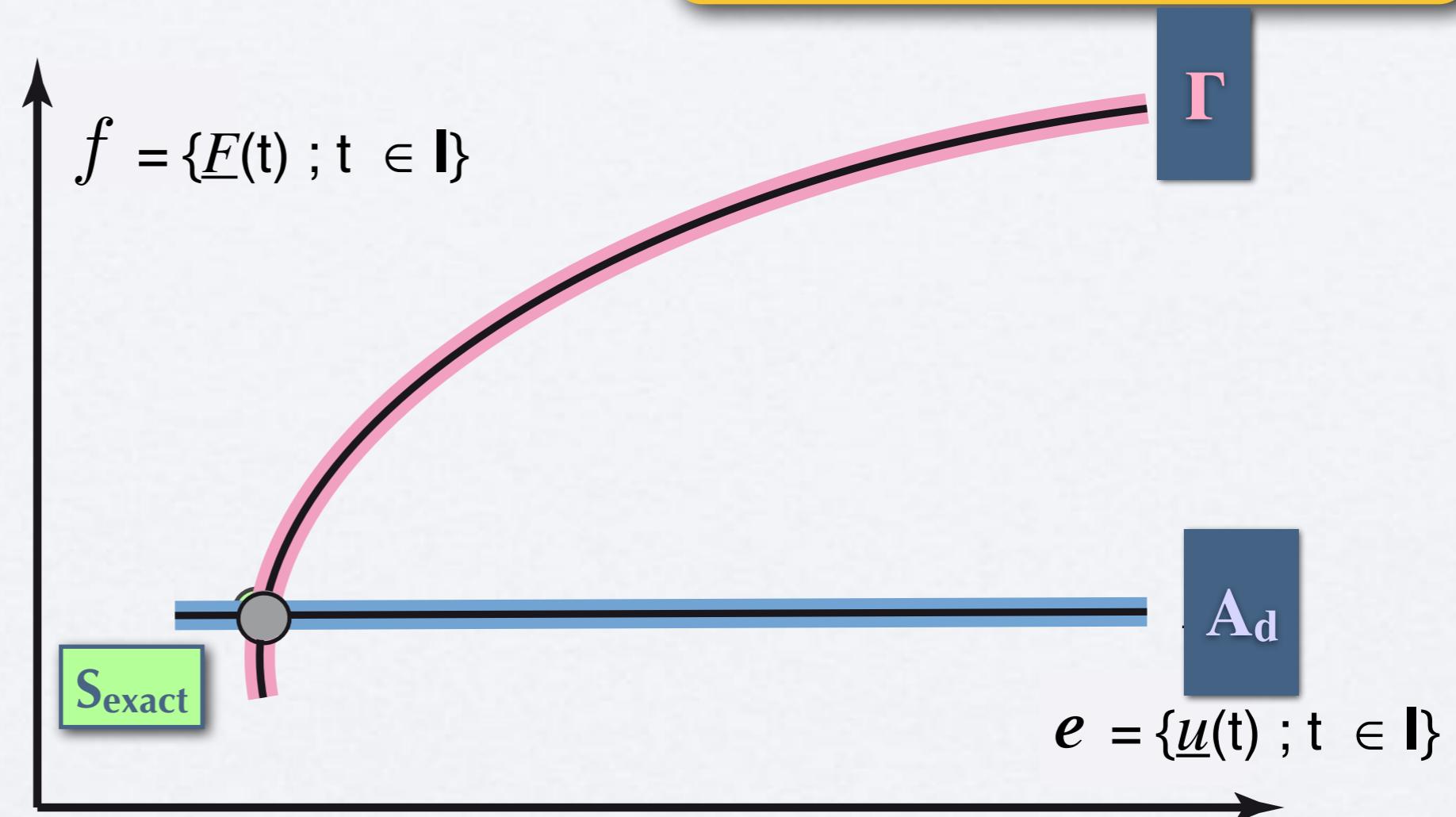
e : element



Reference discretized problem to be solved over $[0, T] \times \Omega$

Global re-formulation

Hypothesis A_d :linear



Reference problem to be solved over $[0, T] \times \Omega$

■ Data : complex loading history $(f_d, F_d, \underline{U}_d)$ over $[0, T]$

Data-ROM:

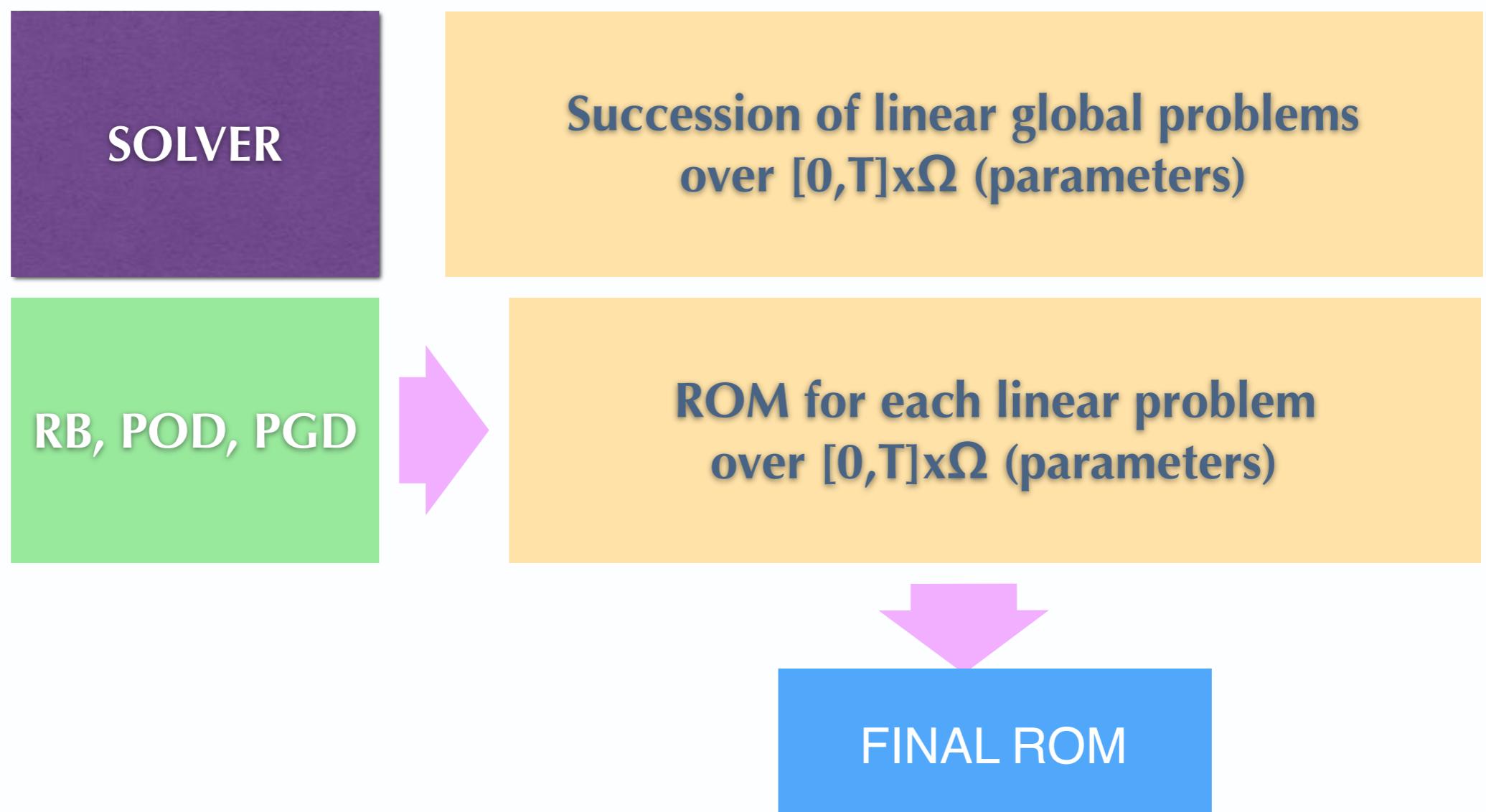
$$\underline{F}_d(t) = \sum_{\ell=1}^r \gamma_\ell(t, \tau) \times \underline{F}_\ell$$
$$\gamma_\ell(t, \tau) = \left(\sum_{k=1}^m \underline{\tau}_k - \text{mode}(H) \right)_\ell$$

Micro-harmonic functions

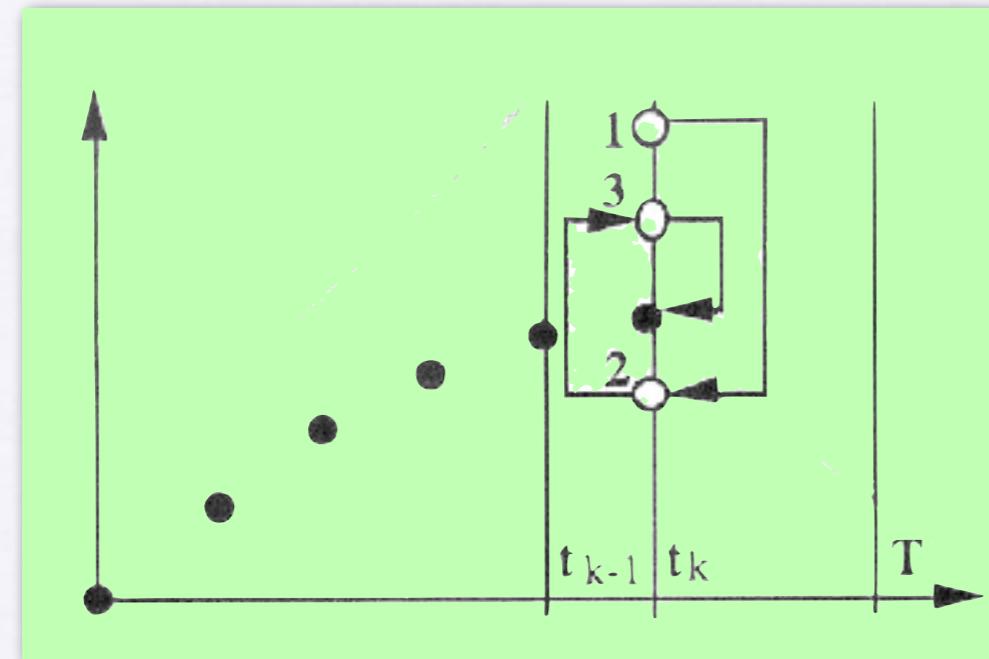
ROM in nonlinear Solid Mechanics



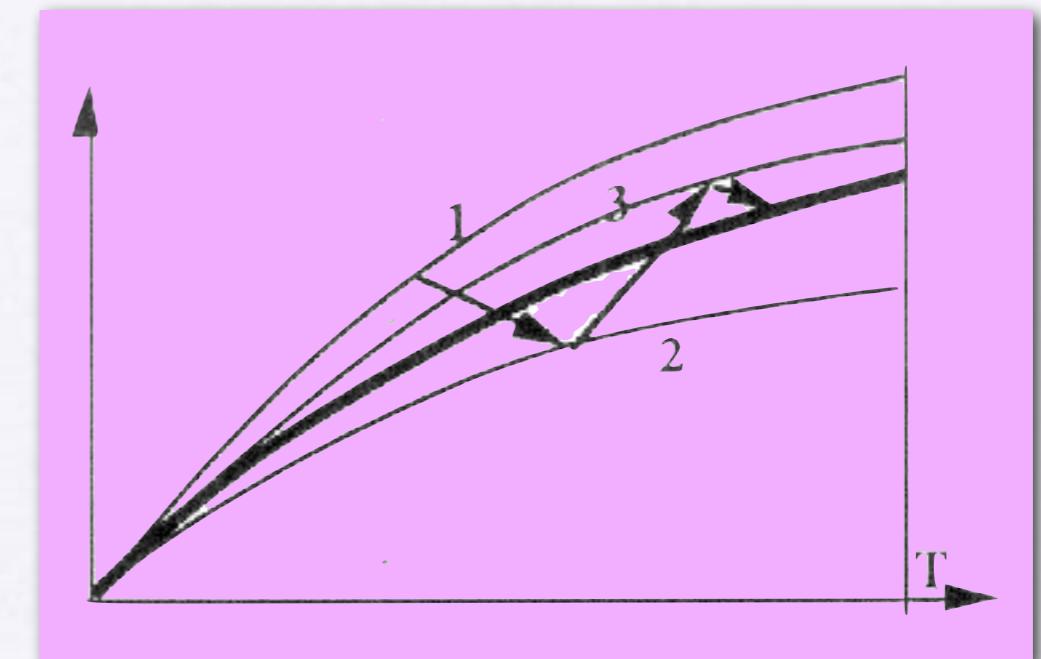
Idea: iterative process (time/space separation)



The solver LATIN



Classical step-by-step methods

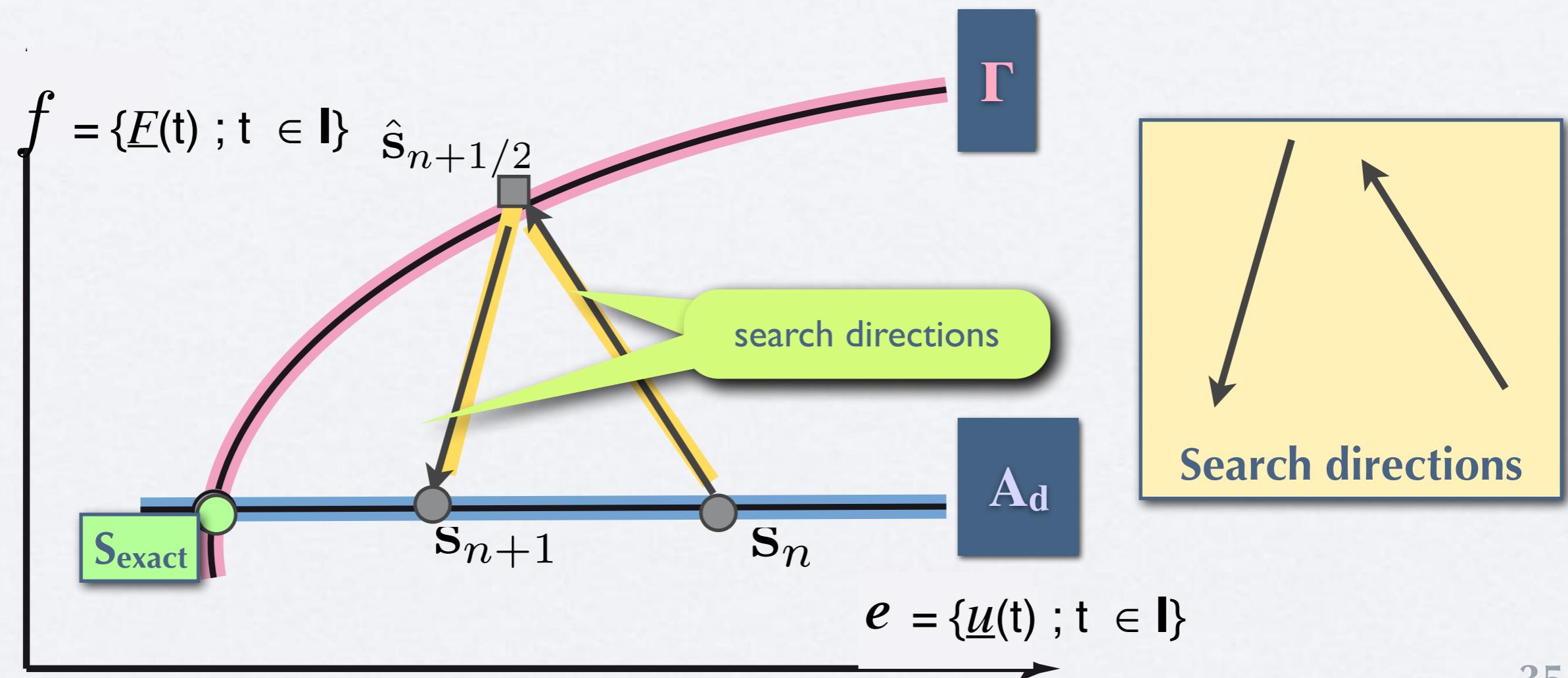
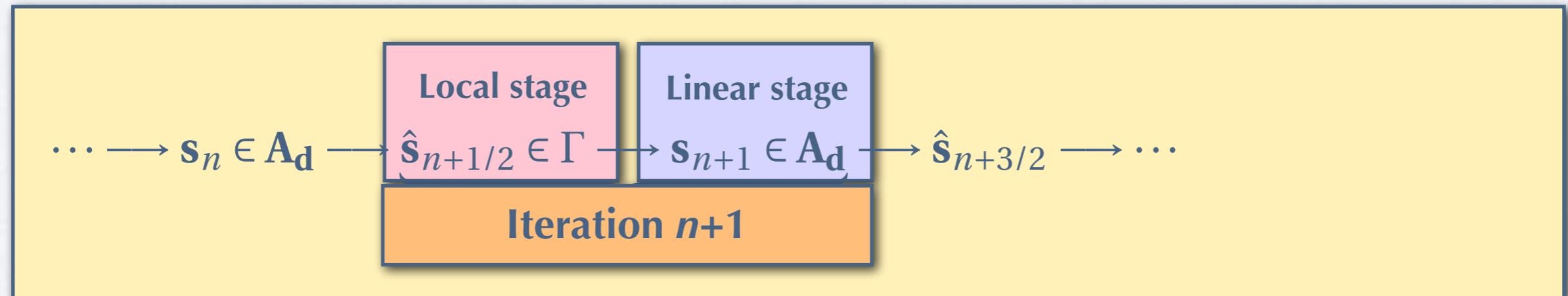


LATIN method

- ❶ **LATIN method: designed for the PGD** (Ladeveze 1985)
- ❷ **LATIN method: mature computational tool**
book [Springer NY 1999]
+ many works

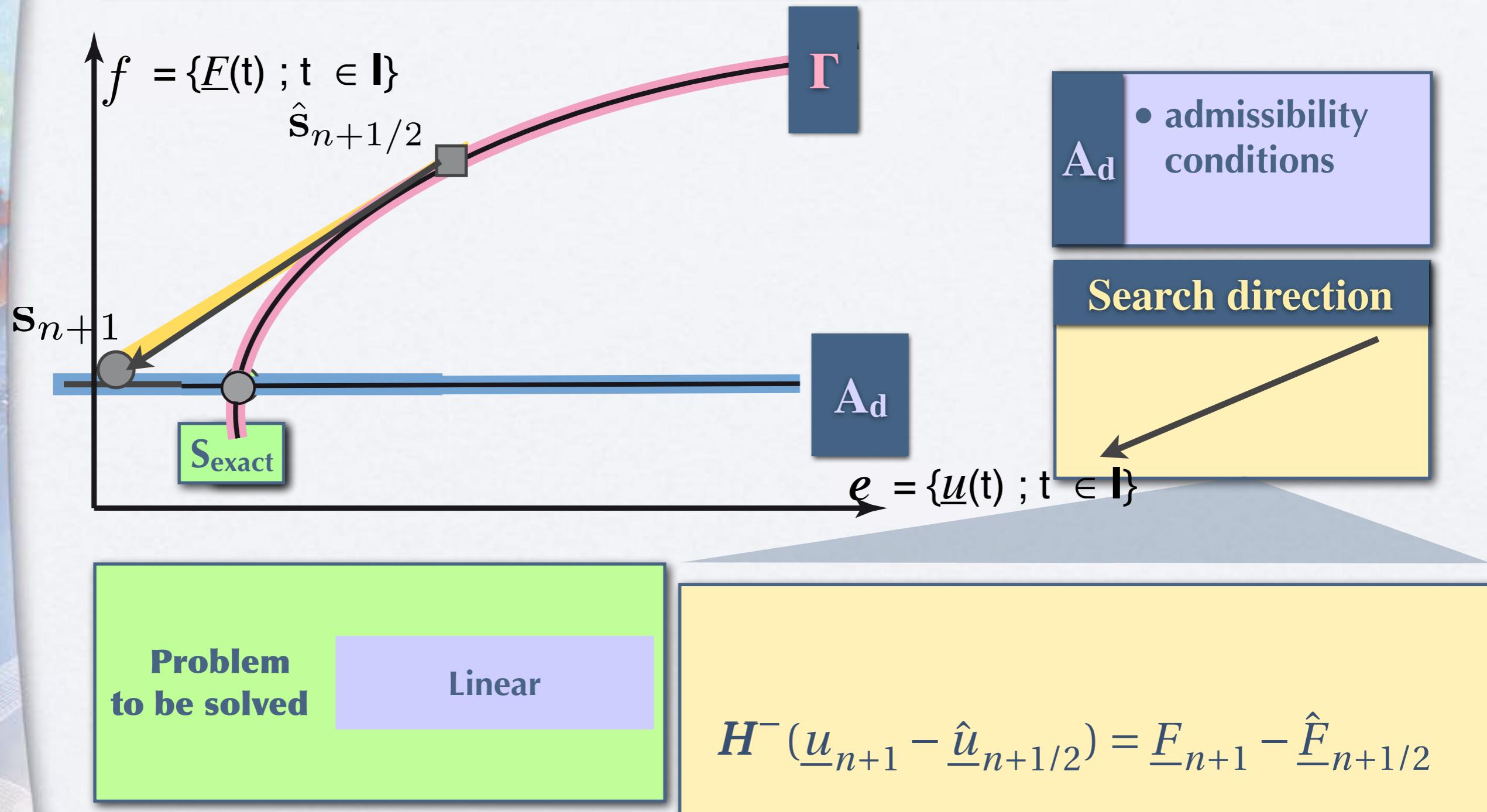
The solver LATIN

Principle :iterative and alternative scheme



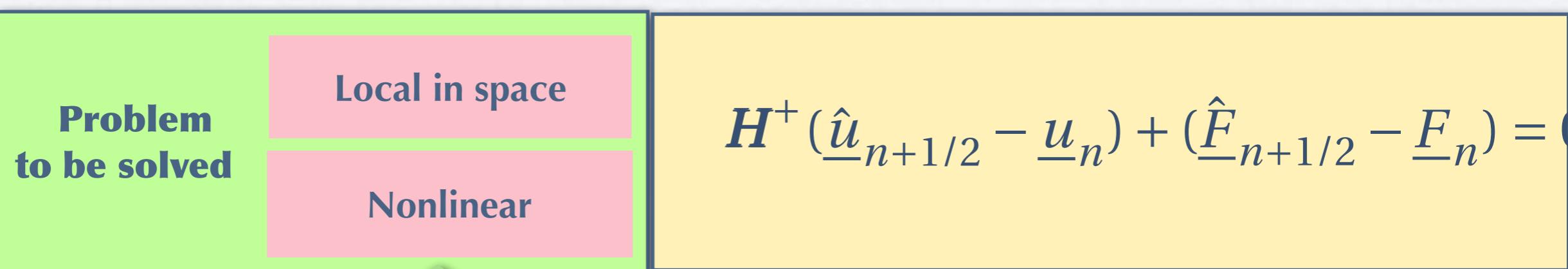
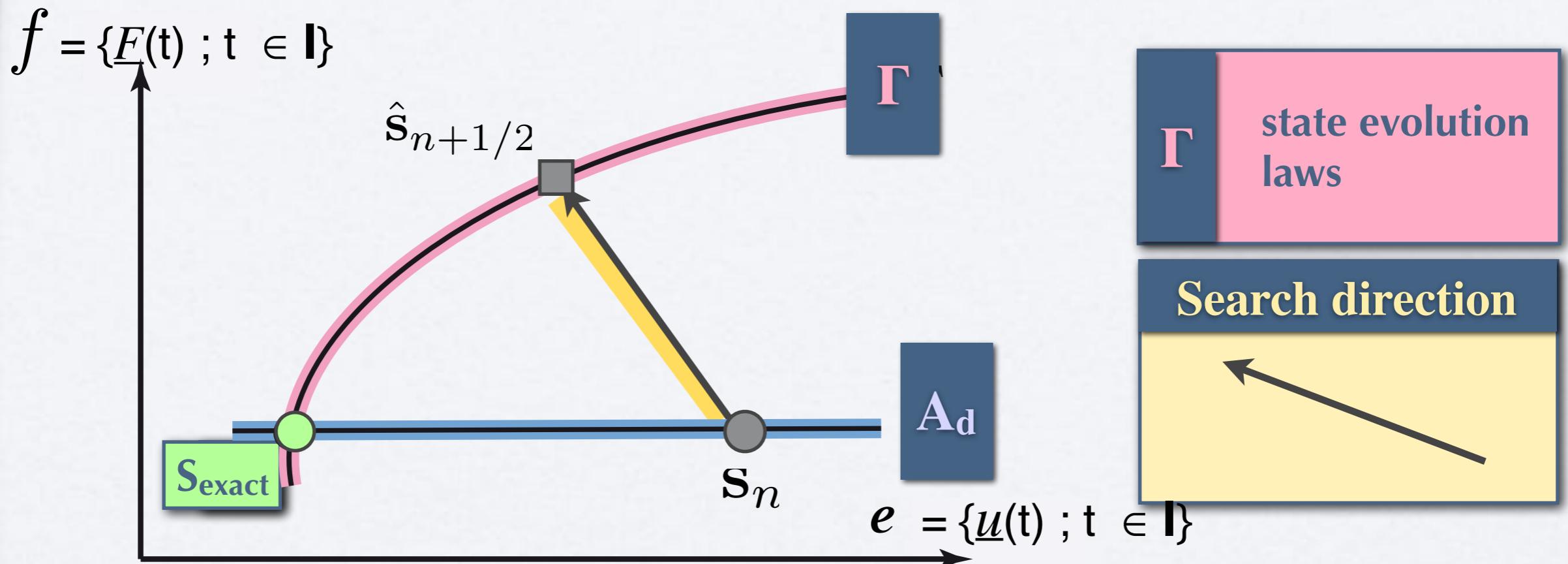
The solver LATIN

Linear stage at Iteration $n+1$: $\hat{S}_{n+1/2} \rightarrow S_{n+1}$



The solver LATIN

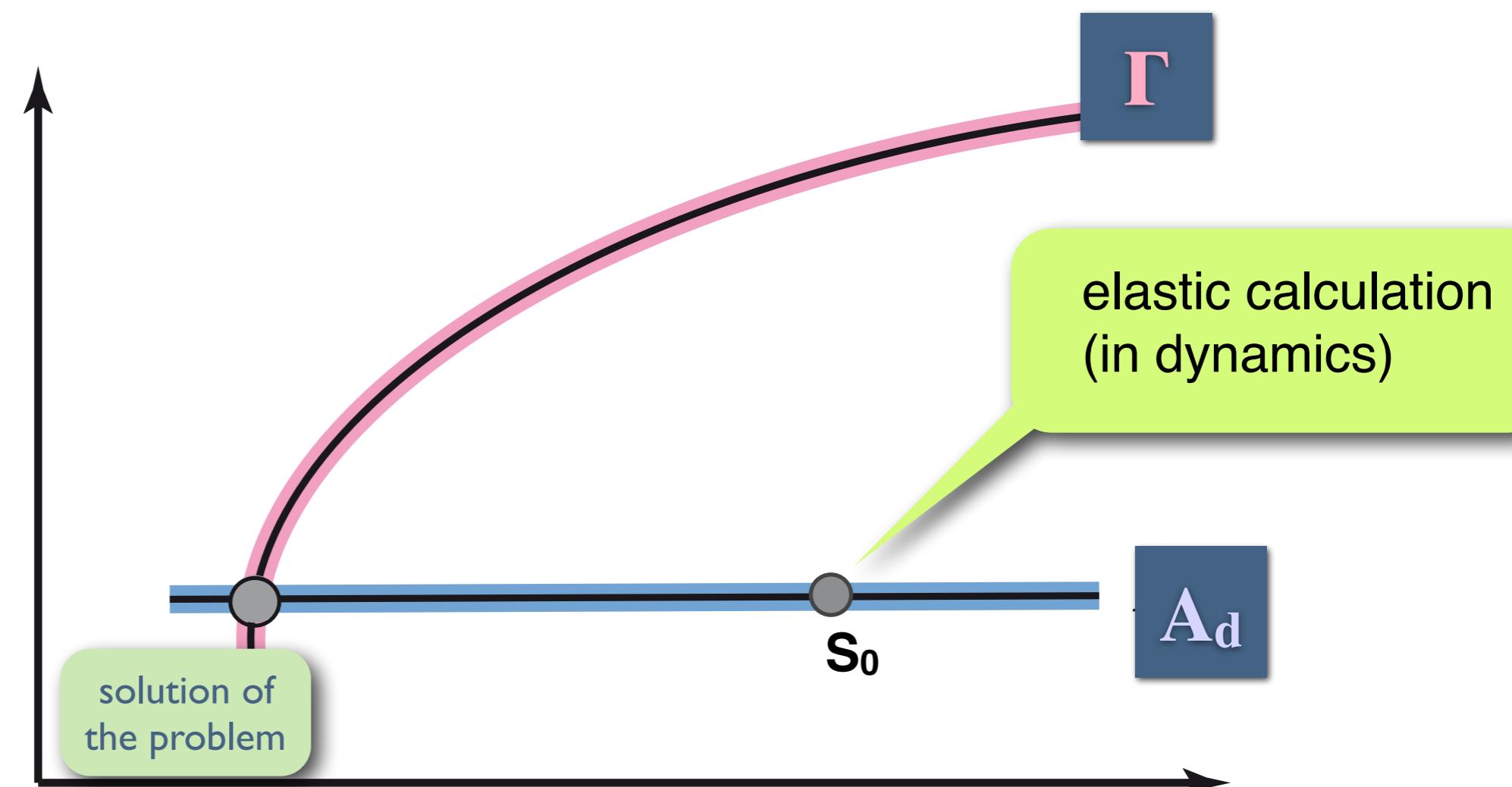
Local stage at Iteration $n+1$: $S_n \rightarrow \hat{S}_{n+1/2}$



very suitable for parallel computing!

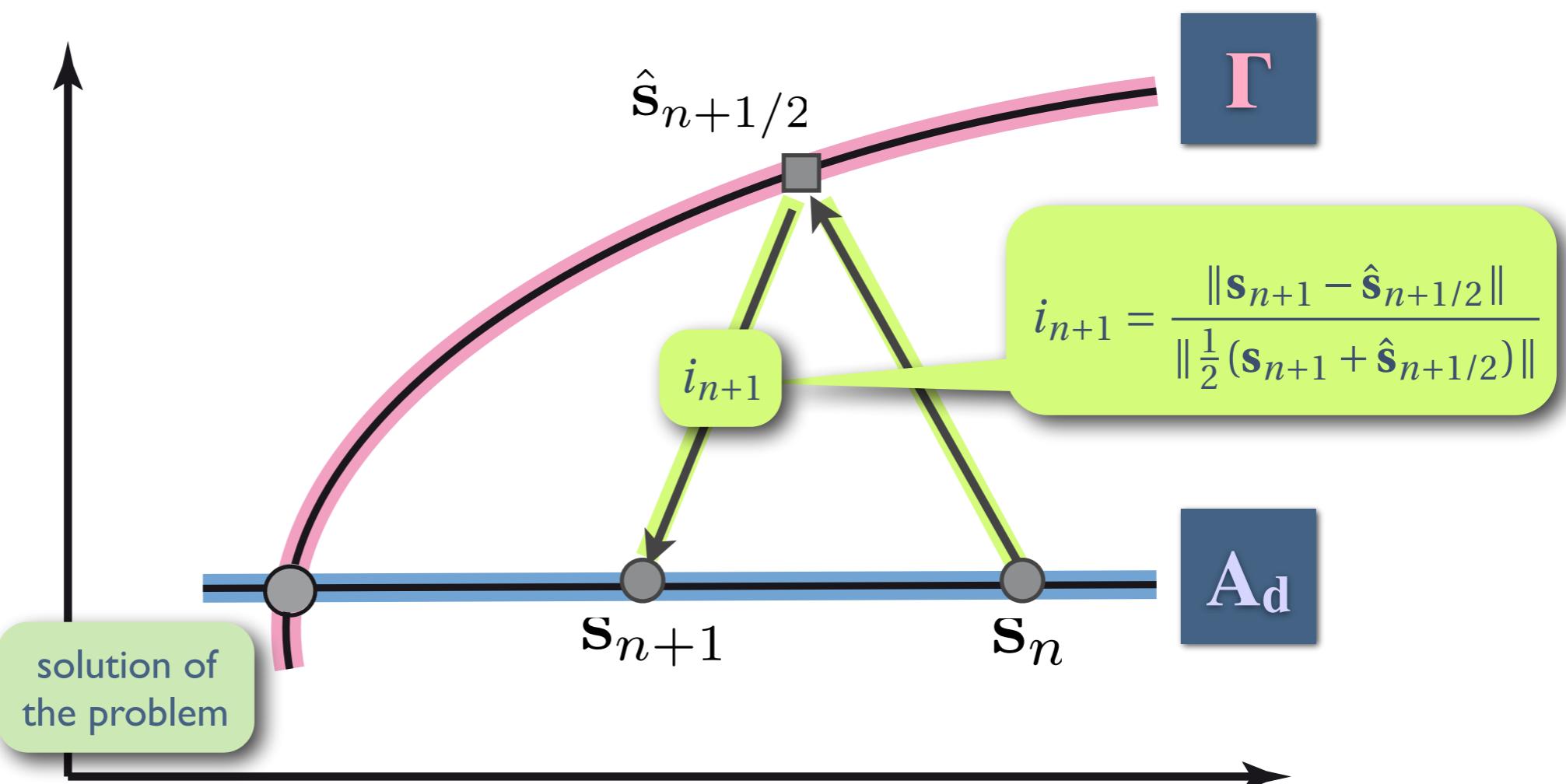
The solver LATIN

Initialisation



The solver LATIN

Error indicator

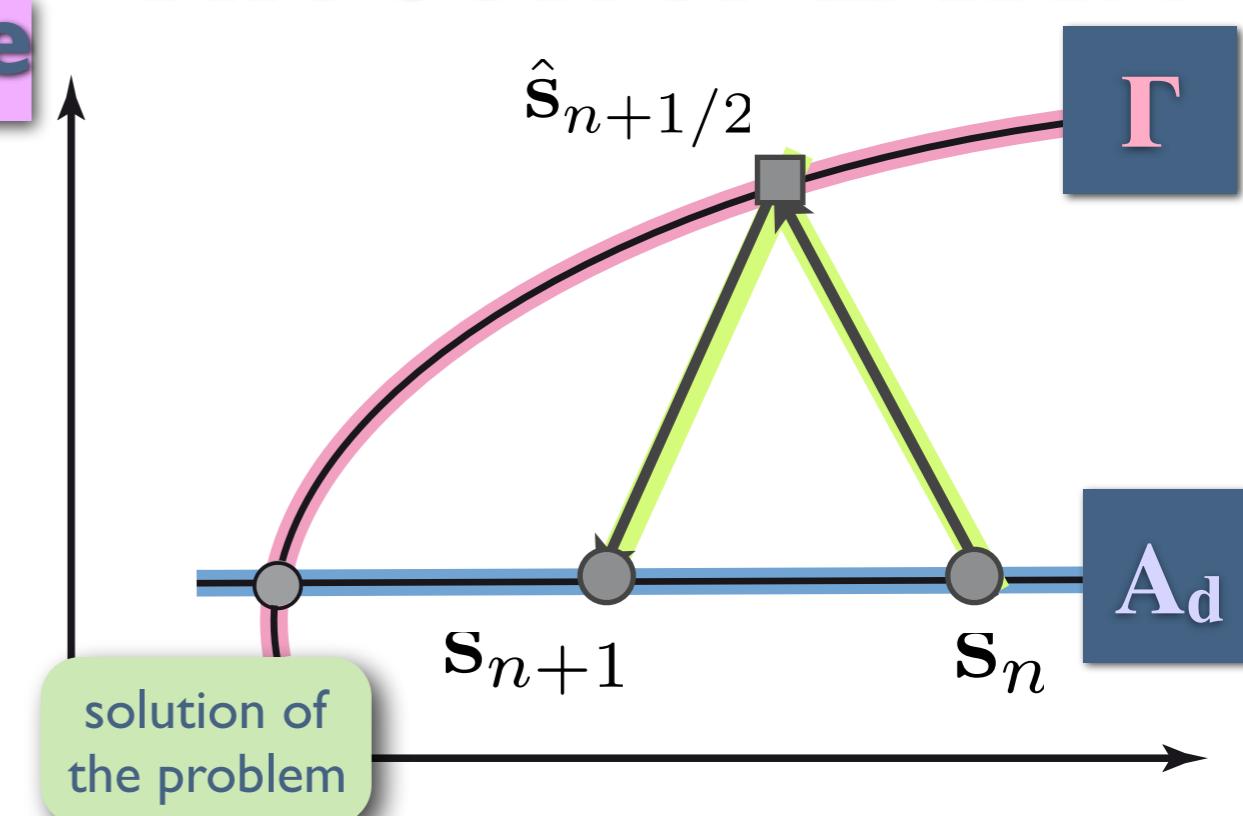


A word about convergence

Dissipation bilinear form:

$$[s, s]_t = \int_0^t \int_{\Omega} f \cdot \dot{e}^p d\Omega dt$$

The solver LATIN



Theorem

Convergence of the LATIN method if

- material operator : monotone
- search directions : $H^+ = H^-$ positive definite,
symmetric

(Ladeveze 1999)

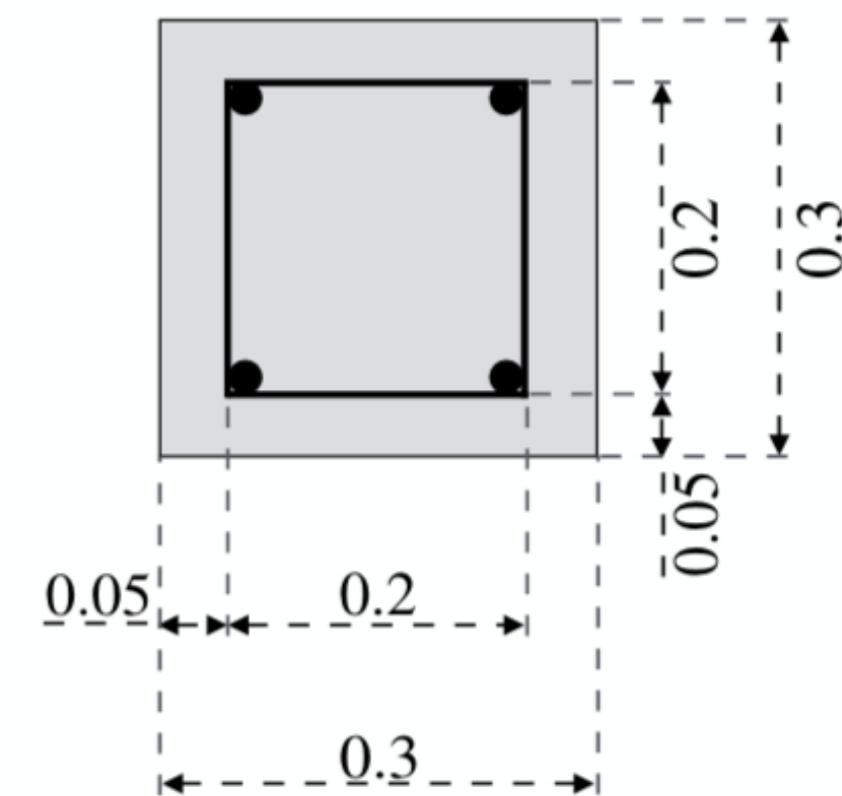
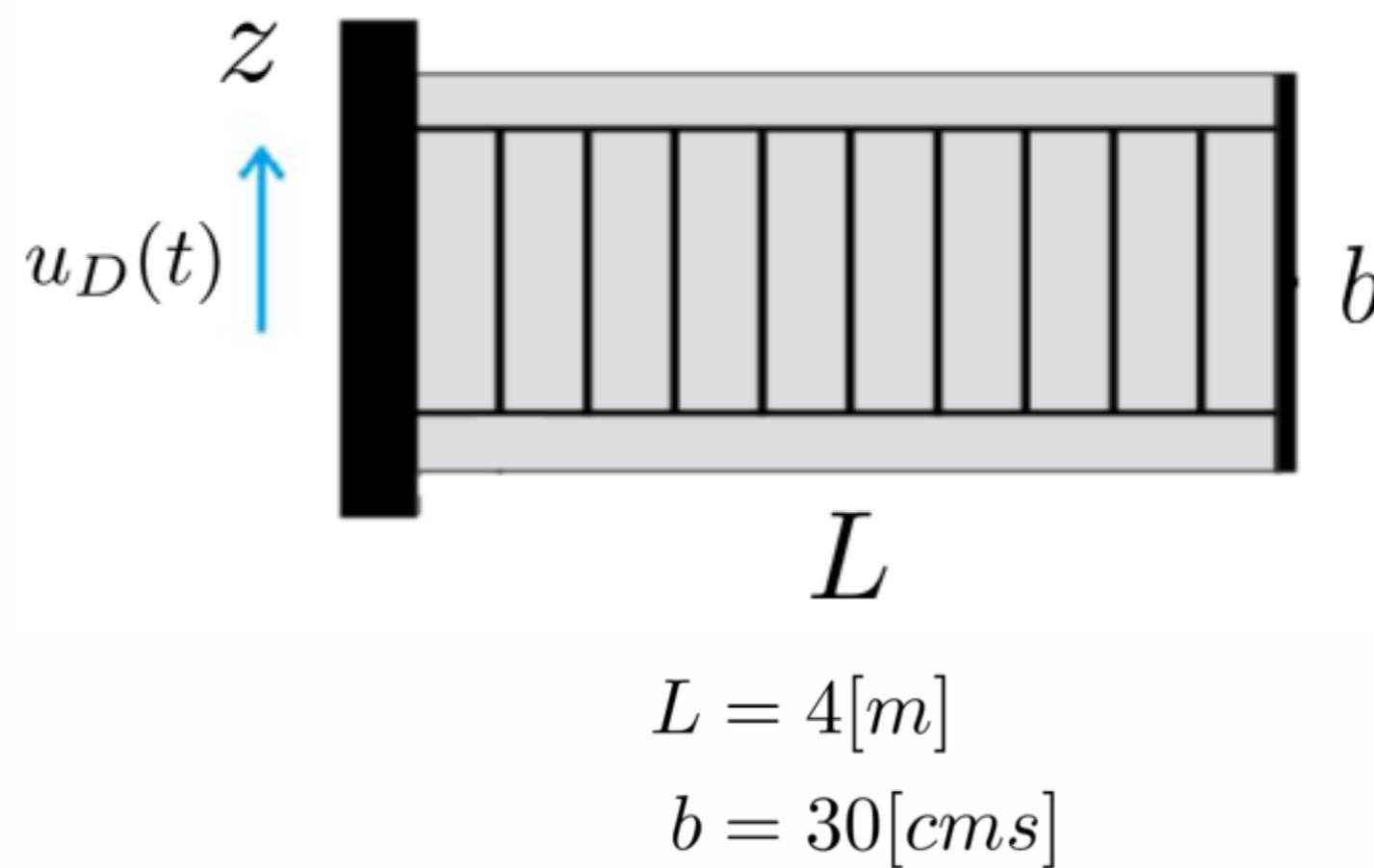
The solver LATIN

Updating of the given force

Linear stage at Iteration $n+1$: $\hat{\mathbf{S}}_{n+1/2} \rightarrow \mathbf{S}_{n+1}$

$F_d(t, \underline{\mathbf{u}}_n(t))$: given

Illustration

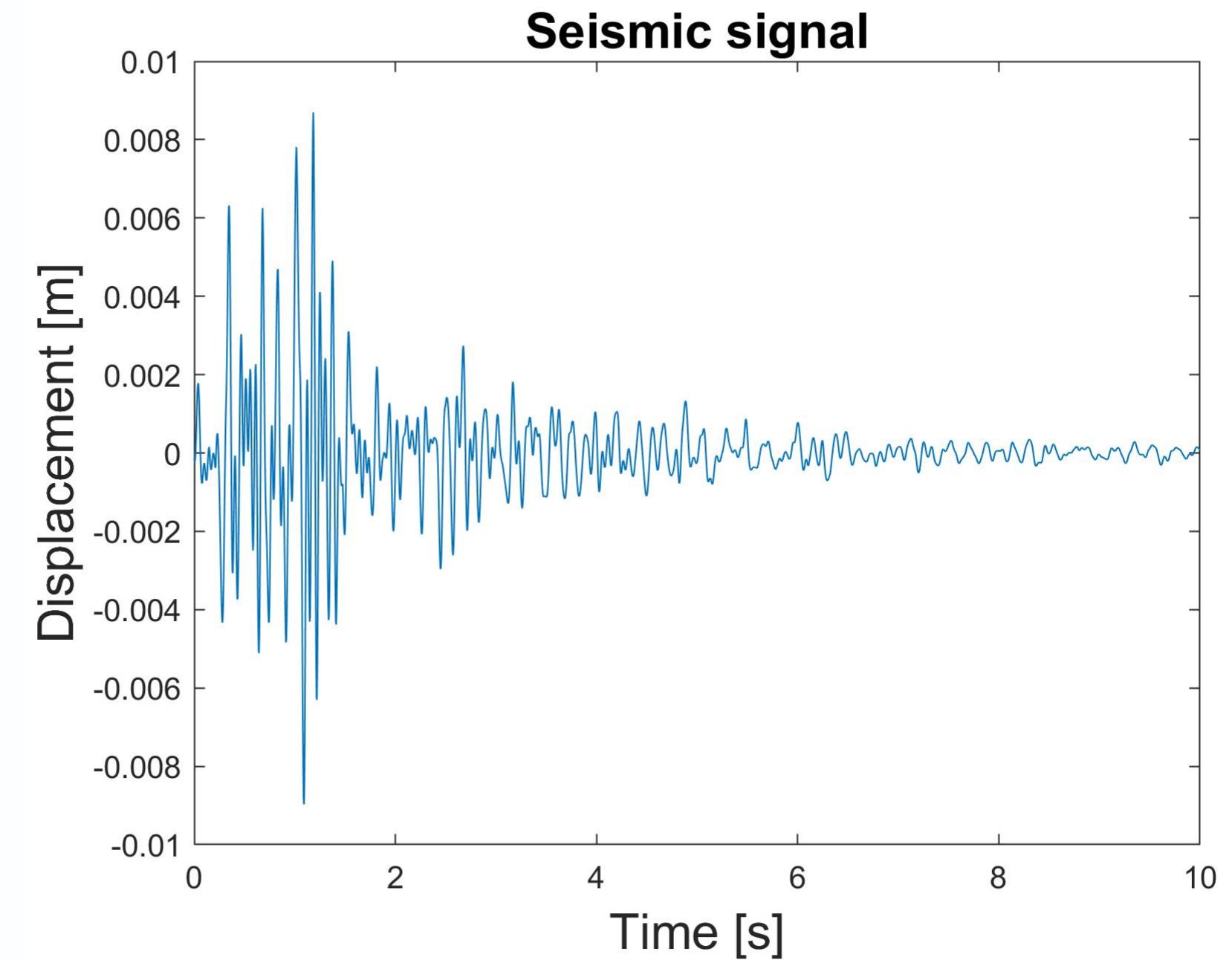


Reinforced concrete structure

Seismic excitation: Imposed displacement at only one side.

Illustration

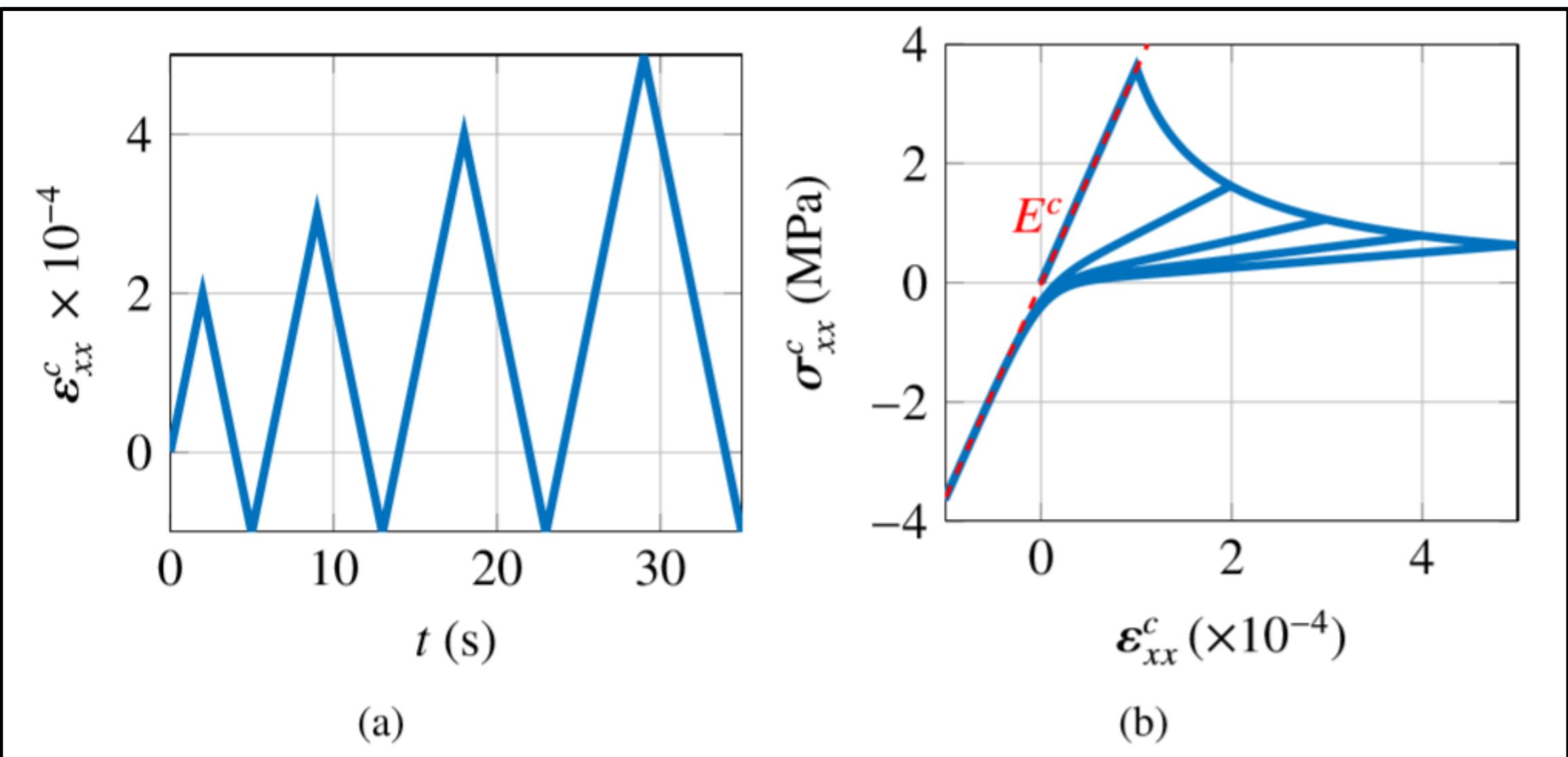
$u_D =$



Approximation with 7 modes (H)-(P1 macro element-error 8%)

Illustration

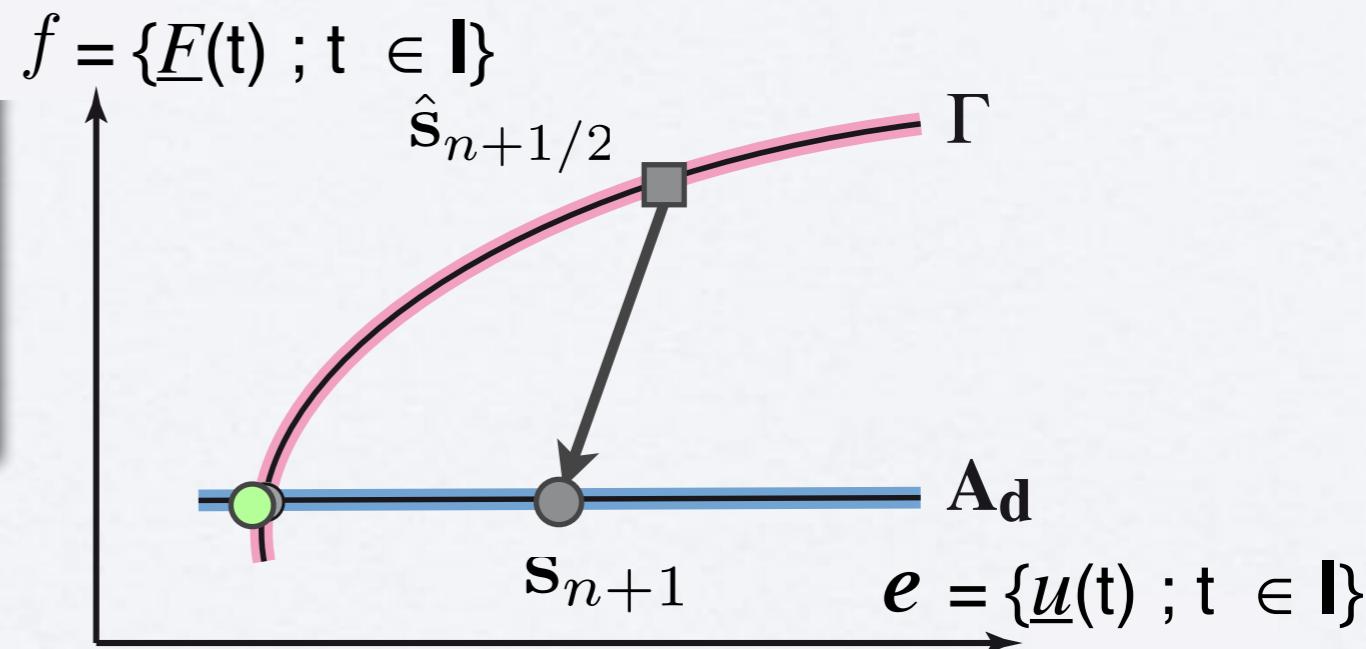
Unilateral damage model (Vassaux et al EFM 2015)



Time-multiscale PGD construction at iteration $n + 1$

Comeback: Linear stage at Iteration $n + 1$: find \mathbf{S}_{n+1}

Problem defined
over $[0, T] \times \Omega$



Find $s(t) = (\underline{u}_{n+1}, \underline{F}_{n+1}) \in \mathcal{S}^{(0, T)}$

$$\underline{F}_{n+1}(t) = \underline{F}_d(t) \quad t \in I$$

$$H^-(\underline{u}_{n+1} - \hat{\underline{u}}_{n+1/2}) = \underline{F}_{n+1} - \hat{\underline{F}}_{n+1/2}$$

Time-multiscale PGD construction at iteration $n+1$

over $[0, T] \times \Omega$

Corrections

$$\Delta \underline{u} \equiv \underline{u}_{n+1} - \underline{u}_n$$

$$\Delta \underline{F} = \underline{F}_{n+1} - \underline{F}_n = 0$$

Find $\Delta \underline{u}(t) \in \mathbf{R}^n \quad t \in I$

$$H^- \Delta \underline{u} = \underline{R}_d = (\underline{F}_d - \hat{\underline{F}}_{n+1/2}) - H^- (\underline{u}_n - \hat{\underline{u}}_{n+1/2})$$

Time-multiscale PGD construction at iteration $n + 1$



Point1 : The PGD-global Residual

Iteration n+1

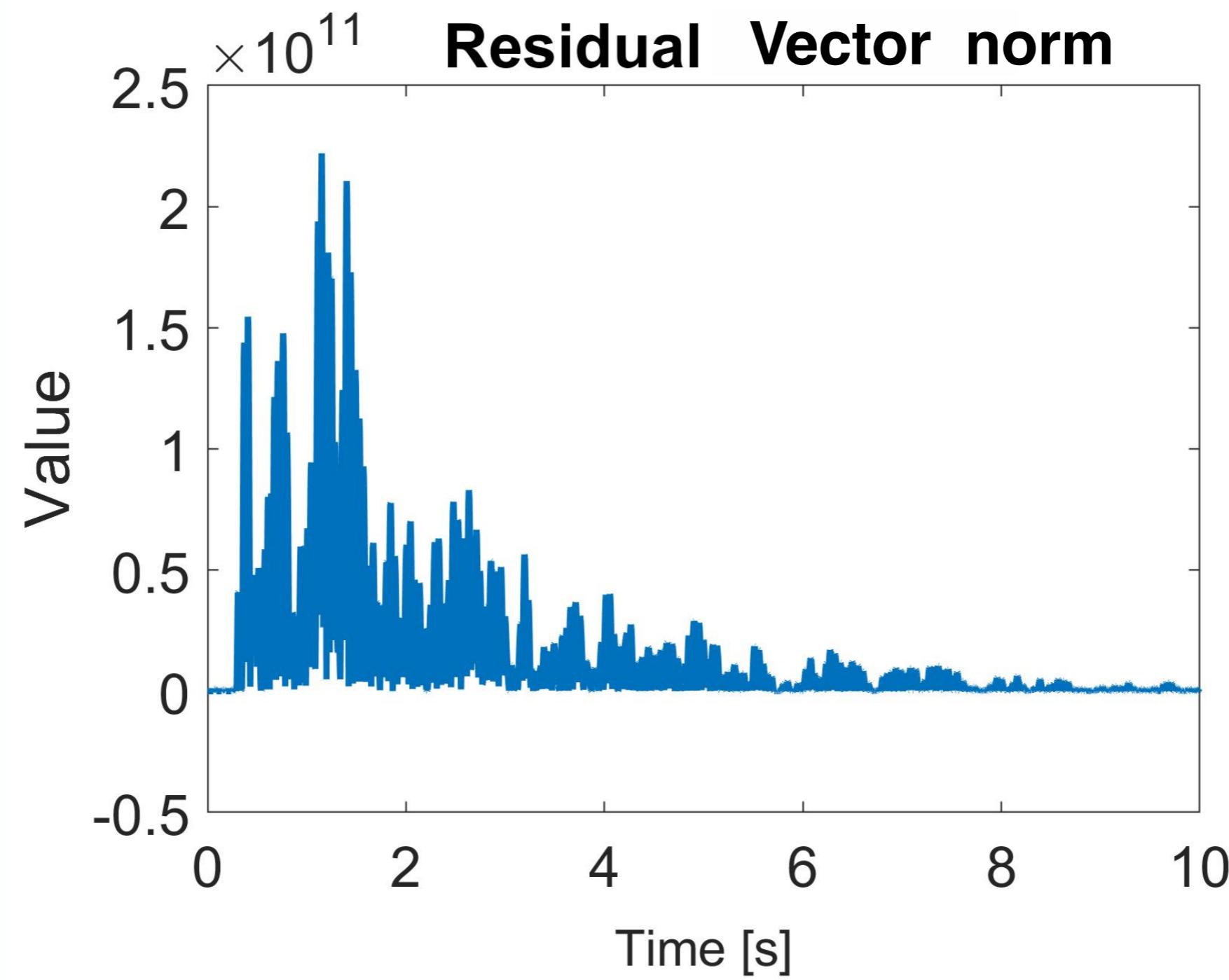
Find $\Delta \underline{u}(t) \in \mathbf{R}^n \quad t \in I$

$$\Delta \underline{u} = \arg \min_{\Delta \underline{u}'} \int_0^T (\mathbf{H}^- \Delta \underline{u}' - \underline{R}_d)^T (\mathbf{H}^-)^{-1} (\mathbf{H}^- \Delta \underline{u}' - \underline{R}_d) dt$$

\underline{R}_d : known

classical PGD $\Delta \underline{u} = \lambda(t) \times \underline{\mathbf{G}}$

Illustration



Time-multiscale PGD construction at iteration $n + 1$



Point 2 : Time-multiscale PGD



Step 1: \underline{R}_d -decomposition

Iteration $n+1$

$$\underline{R}_d : \sum_{k=1}^r \underline{R}_d^k$$



$$\Delta \underline{u} : \sum_{k=1}^r \Delta \underline{u}^k$$

$$\underline{R}_d^k : \tau_k - \text{mode } (P) \times \underline{\mathbb{Z}}^k$$

● Hypothesis : Period $\tau_k = \underline{\text{given loading period }} \underline{\tau}_k$

Time-multiscale PGD construction at iteration $n + 1$



Point 2 : Time-multiscale PGD

Step 1: \underline{R}_d -decomposition

$$\underline{R}_d^k = \underline{n}^k(\tau, \underline{\tau}_k) S \underline{A}^k \times \underline{Z}^k$$

Extension of the signal theory

with $\underline{n}^k(\tau, \underline{\tau}_k) = \begin{bmatrix} h_R^k(\tau, \underline{\tau}_k) & (\text{even}) \\ h_I^k(\tau, \underline{\tau}_k) & (\text{odd}) \end{bmatrix}$

$$\int_{-\underline{\tau}_k/2}^{\underline{\tau}_k/2} (h_R^k)^2 d\tau = \int_{-\underline{\tau}_k/2}^{\underline{\tau}_k/2} (h_I^k)^2 d\tau = \underline{\tau}_k/2$$

$$h_I^k(\underline{\tau}_k/2) = 0$$

Unknowns : $\underline{Z}^k, h_R^k, h_I^k, \underline{A}^k$

Time-multiscale PGD construction at iteration $n + 1$



Point 2 : Time-multiscale PGD

Step 1: \underline{R}_d -decomposition $\underline{R}_d^k = \underline{n}^k(\tau, \tau_k) \underline{S} \underline{A}^k \times \underline{Z}^k$

Unknowns : $\underline{Z}^k, h_R^k, h_l^k, \underline{A}^k$

$$\underline{R}_d^k = \arg \min_{\underline{R}_d^{k'}} \frac{1}{T} \int_0^T (\underline{R}_d - \underline{R}_d^{k'}) \cdot (\underline{R}_d - \underline{R}_d^{k'}) dt$$

Time-multiscale PGD construction at iteration $n + 1$



Point 2 : Time-multiscale PGD

◆ Step 1: \underline{R}_d -decomposition $\underline{R}_d^k = \underline{n}^k(\tau, \tau_k) S \underline{A}^k \times \underline{Z}^k$

$$(\underline{A}^k, h_R^k, h_I^k) = \arg \max_{\underline{A}, h_R, h_I} \frac{\underline{A}^T \langle \tilde{S}^T \underline{n} \underline{R}_d^T \rangle \langle \tilde{\underline{R}}_d \underline{n}^T \tilde{S} \rangle \underline{A}}{\underline{A}^T \mathbb{M} \underline{A}}$$

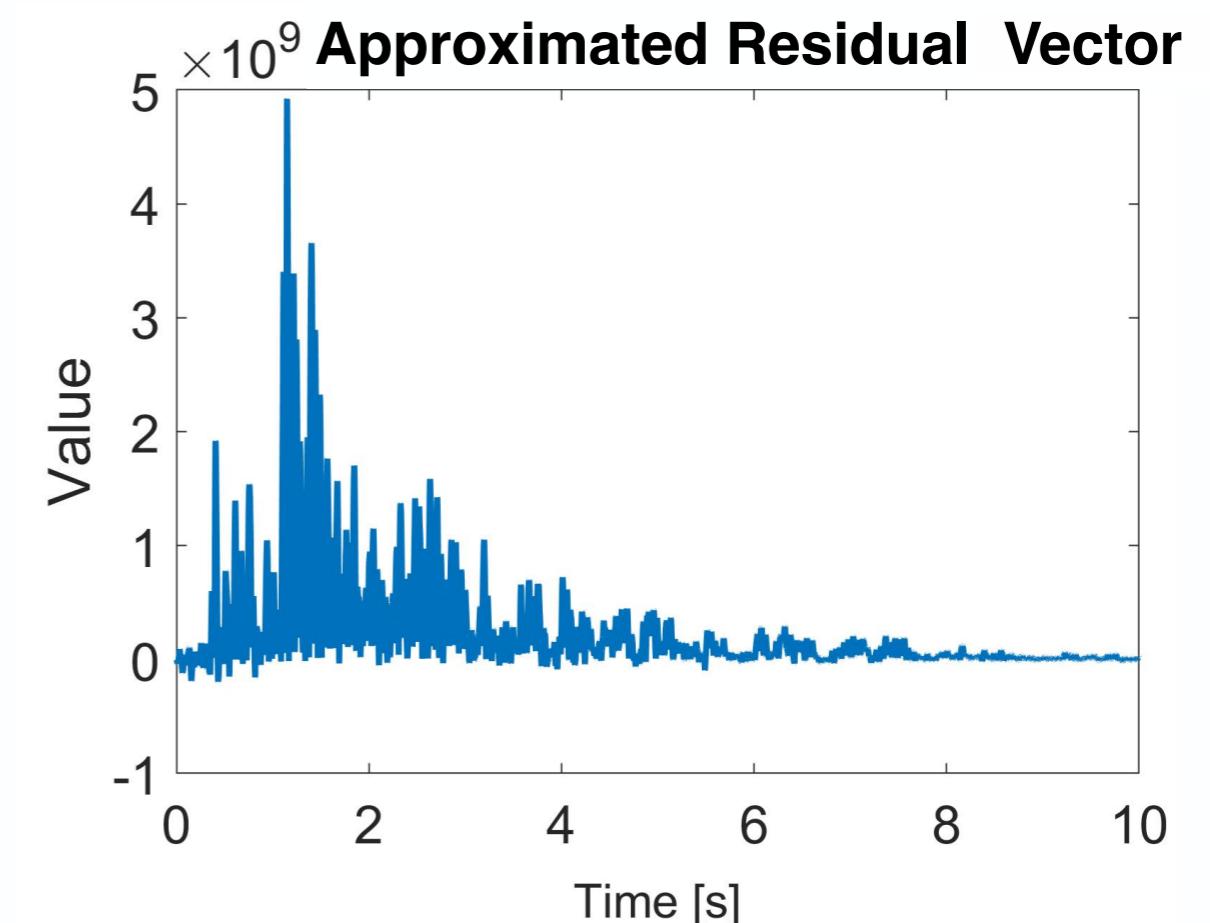
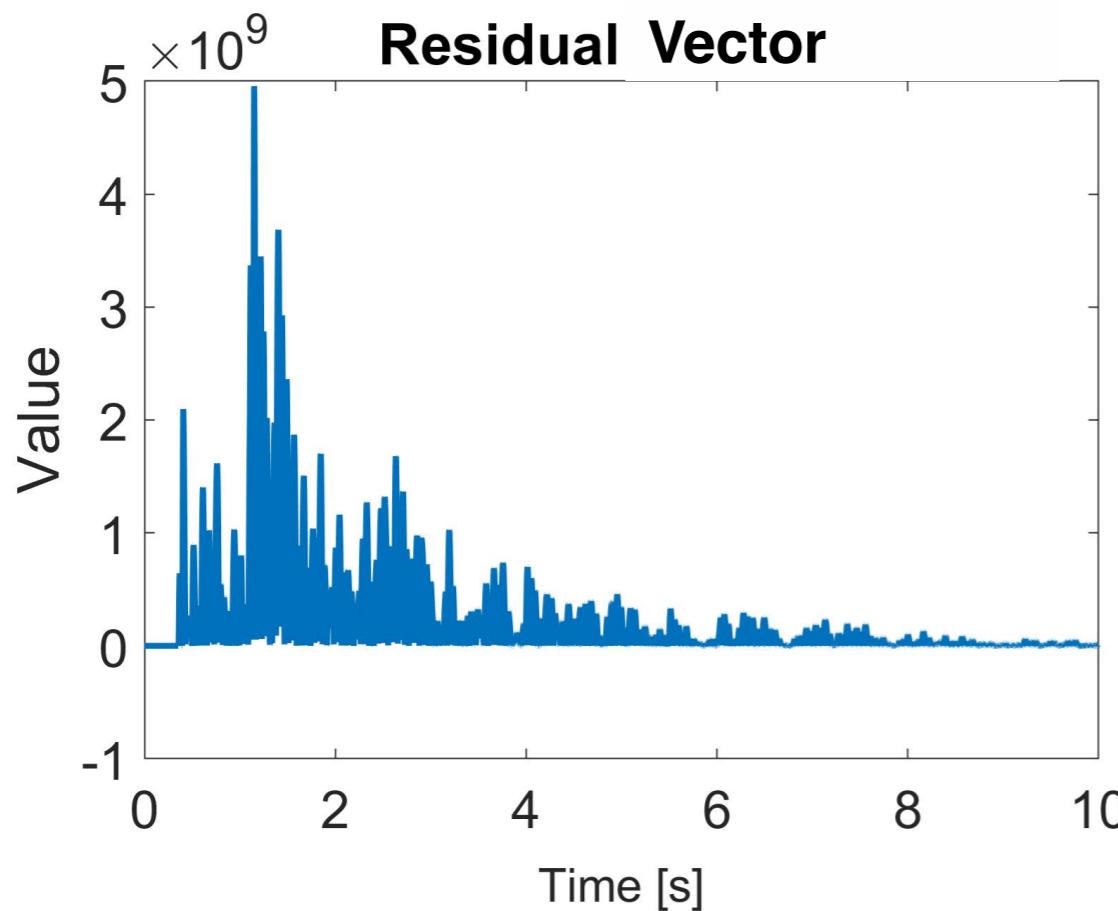
$$\tilde{f}(\tau) = \sum_{j=1}^m f(t_j + \tau)$$

$$\langle \bullet \rangle = \int_{-\underline{\tau}_k/2}^{\underline{\tau}_k/2} \bullet d\tau$$

$$\underline{Z}^k = \langle \underline{n}^k(\tau, \underline{\tau}_k) \tilde{S} \underline{A}^k \times \tilde{\underline{R}}_d \rangle / \underline{A}^{kT} \mathbb{M} \underline{A}^k$$

Illustration

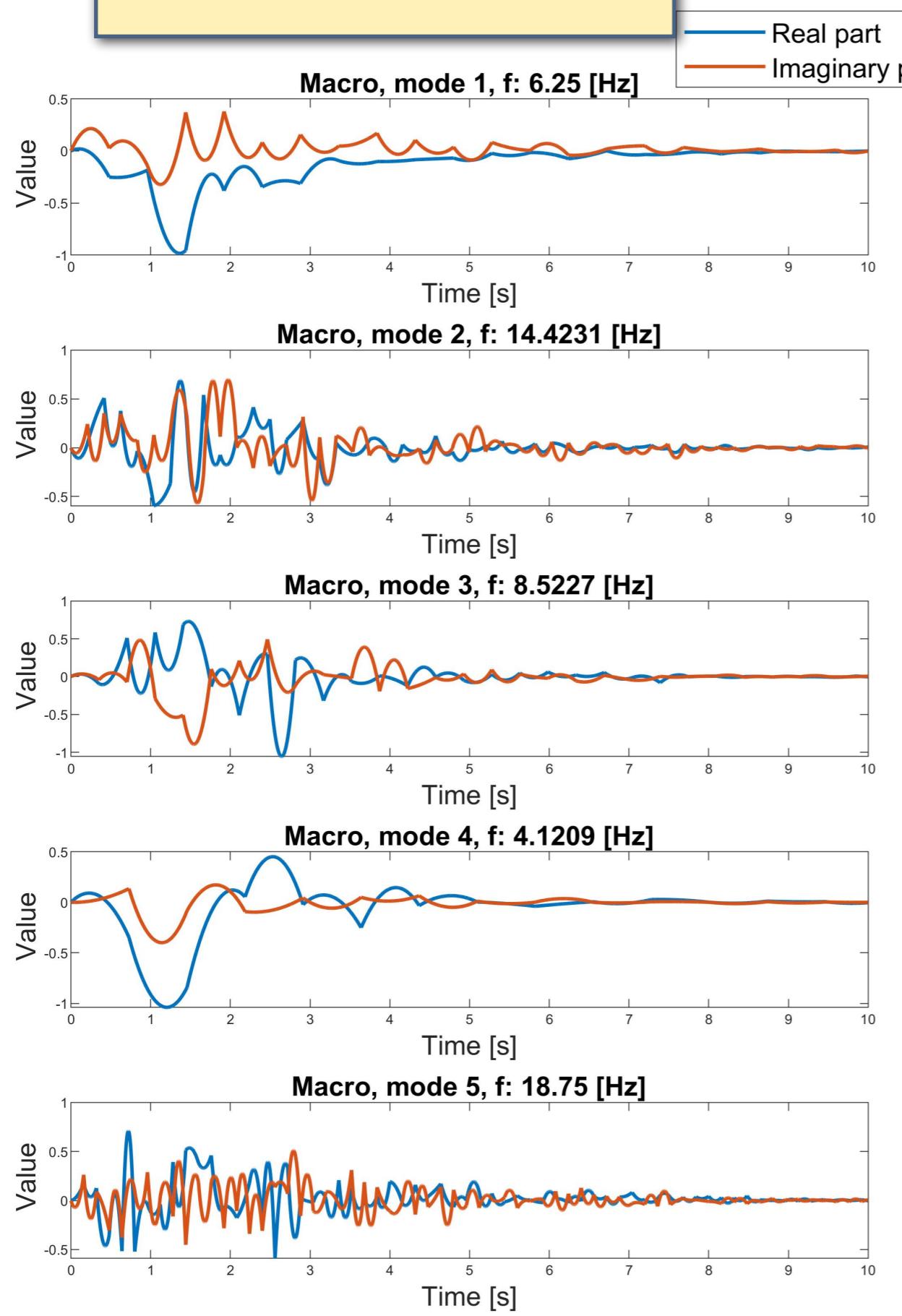
Error : 4,8%



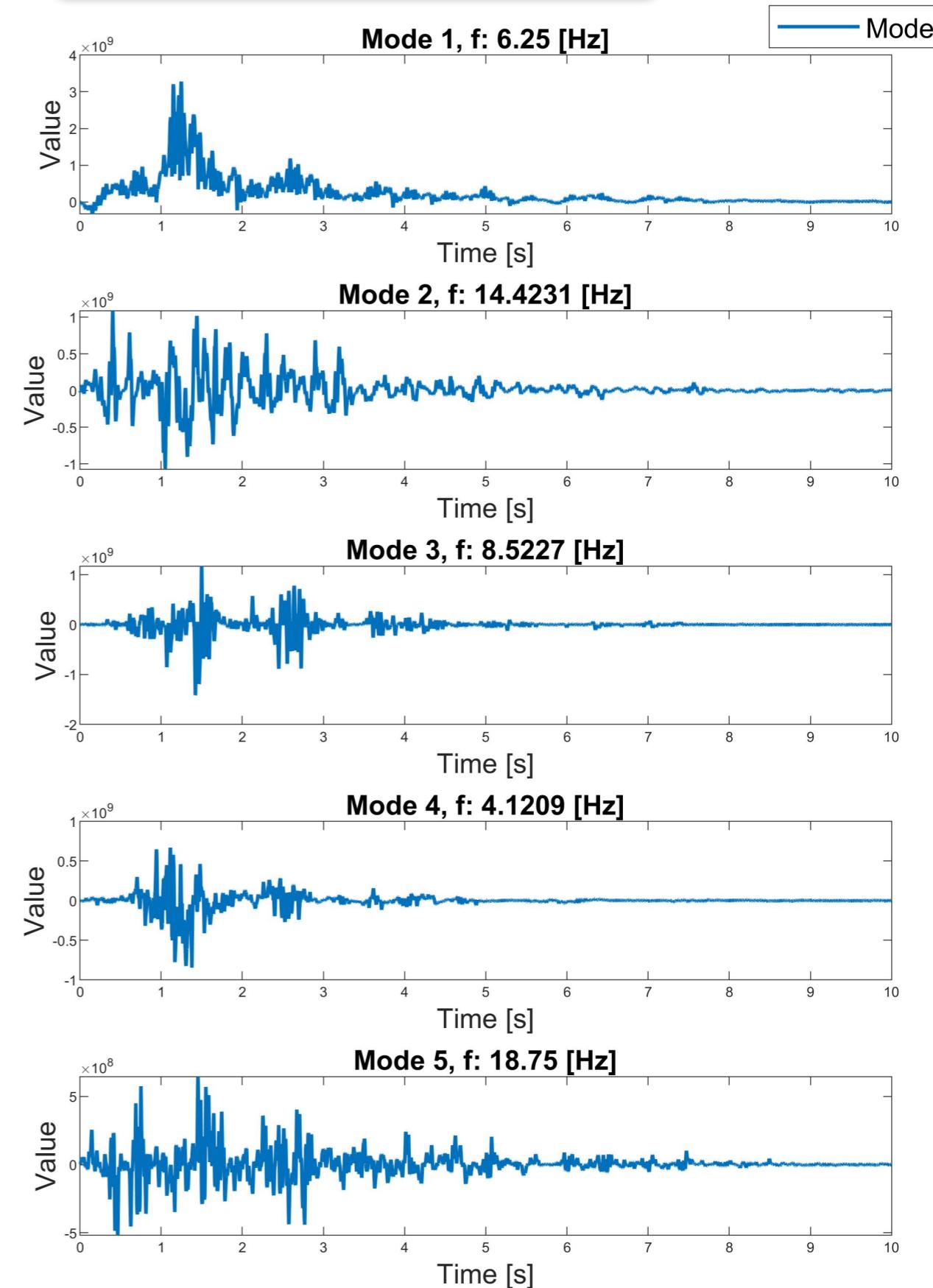
increases a little bit the number of iterations

Illustration

Macro-amplitudes



Residual modes



Time-multiscale PGD construction at iteration $n + 1$

Iteration $n+1$



Point 2 : Time-multiscale PGD

◆ Step 2 : Two time-scales PGD computation of $\Delta \underline{u}^k$

$$\Delta \underline{u}^k = \underline{T}_k - \text{mode } (\underline{P}) \times \underline{g}^k \text{ with } \underline{h}_R^k, \underline{h}_I^k$$

$$\underline{R}_d^k$$

Unknowns : $\underline{g}^k, \underline{A}^k$

Time-multiscale PGD construction at iteration $n + 1$



Point 2 : Time-multiscale PGD



Step 2 : Two time-scales PGD computation of $\Delta \underline{u}^k$

Iteration $n+1$

Find

$\Delta \underline{u}^k$

$$\Delta \underline{u}^k = \arg \min_{\Delta \underline{u}'^k} \int_0^T dt [H^- \Delta \underline{u}'^k - \underline{R}_d^k]^T H^{-1} [H^- \Delta \underline{u}'^k - \underline{R}_d]$$

Time-multiscale PGD construction at iteration $n + 1$

Iteration $n+1$

Point 2 : Time-multiscale PGD

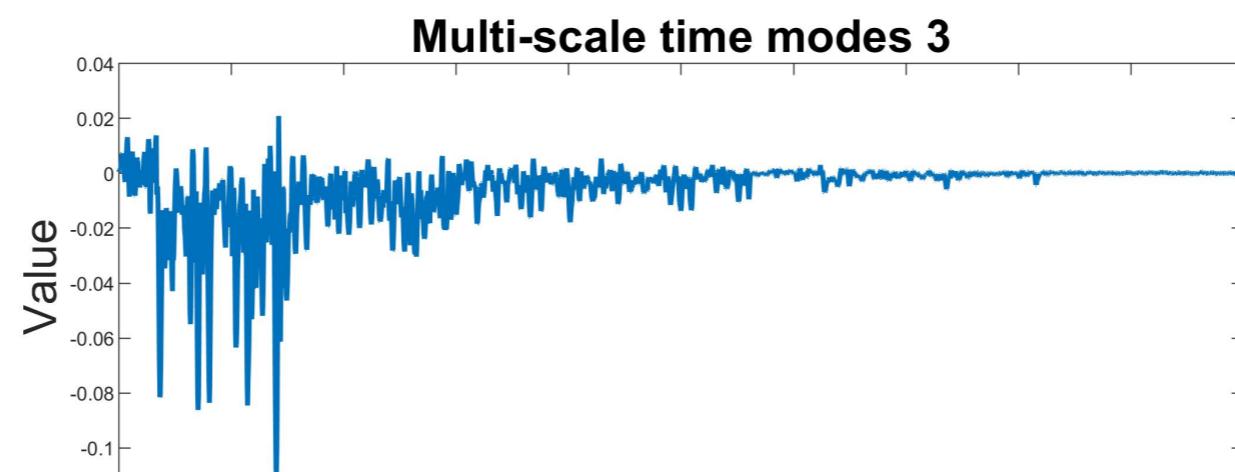
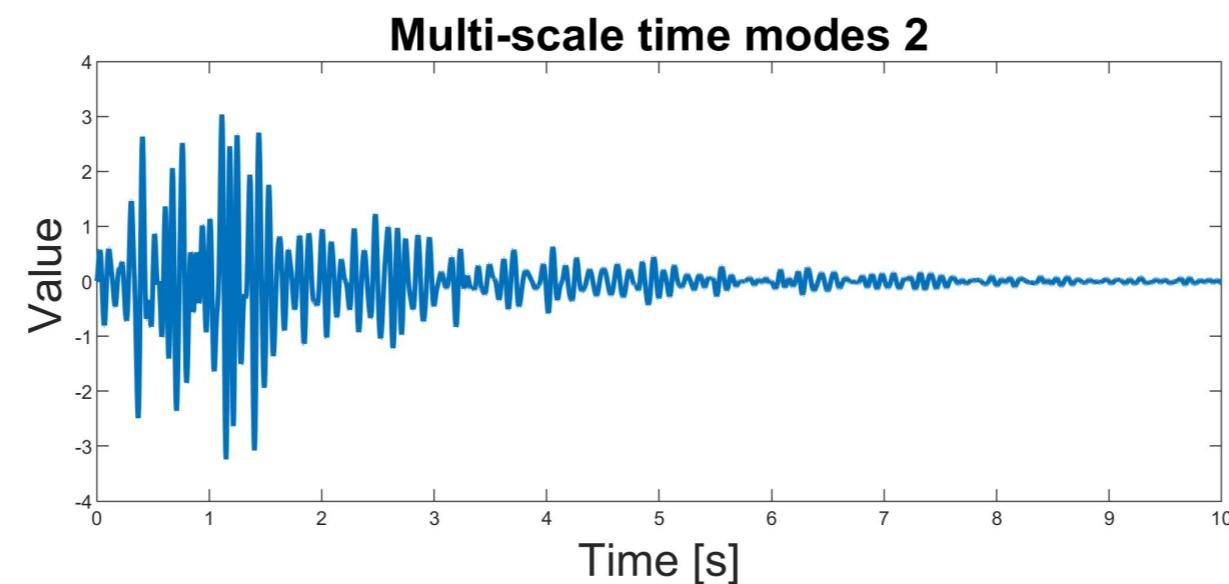
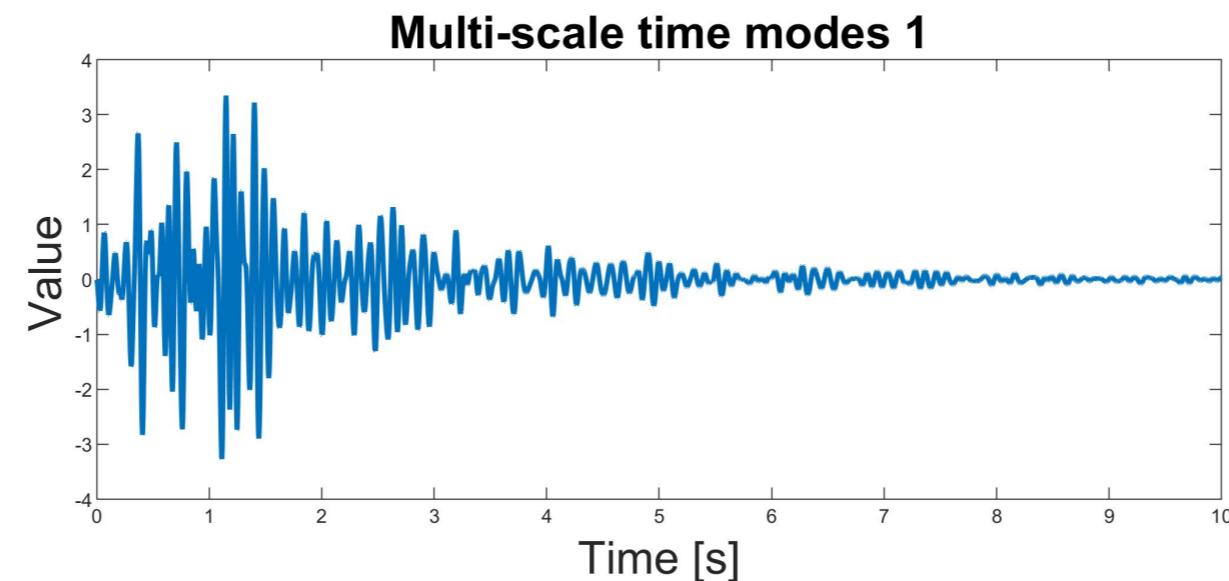
Step 2 : Two time-scales PGD computation of $\underline{\Delta u}^k$

• **Principle : alternative minimization on time functions and space functions (greedy calculation)**

- time-problem: $(\underline{\mathbf{A}}^k)$
- space -problem: standard FE problem ($\underline{\mathbf{g}}^k$)

• **Final step :** $\underline{\Delta u} = \sum_{k=1}^m \underline{\Delta u}^k$

Illustration



Outline

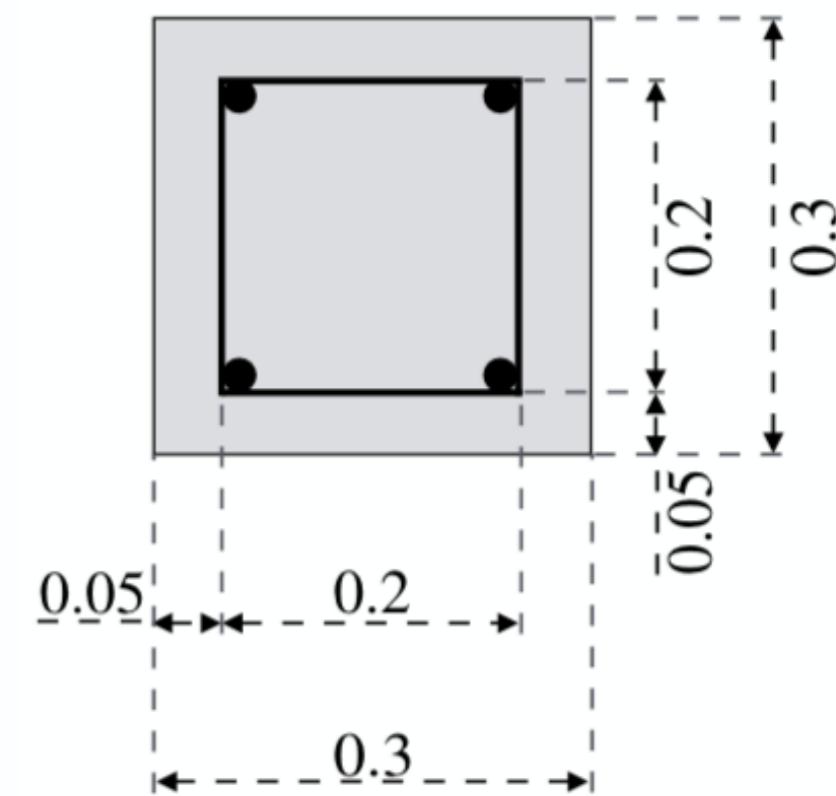
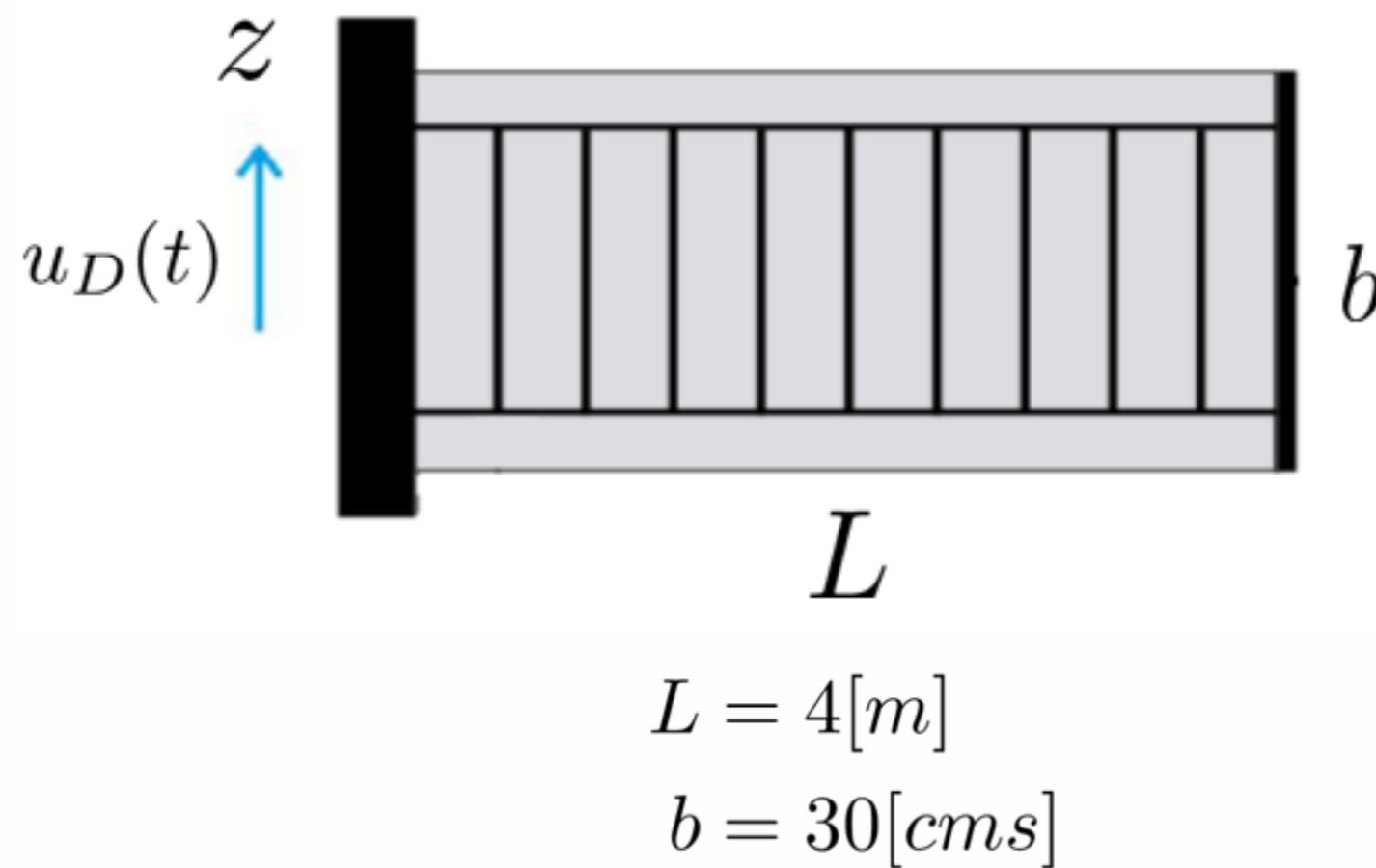
1. A signal theory :a ROM for the loading

2. A new PGD approach : multiscale in time and non intrusive

3. First illustrations

4. Conclusion-Prospects

Seismic illustration

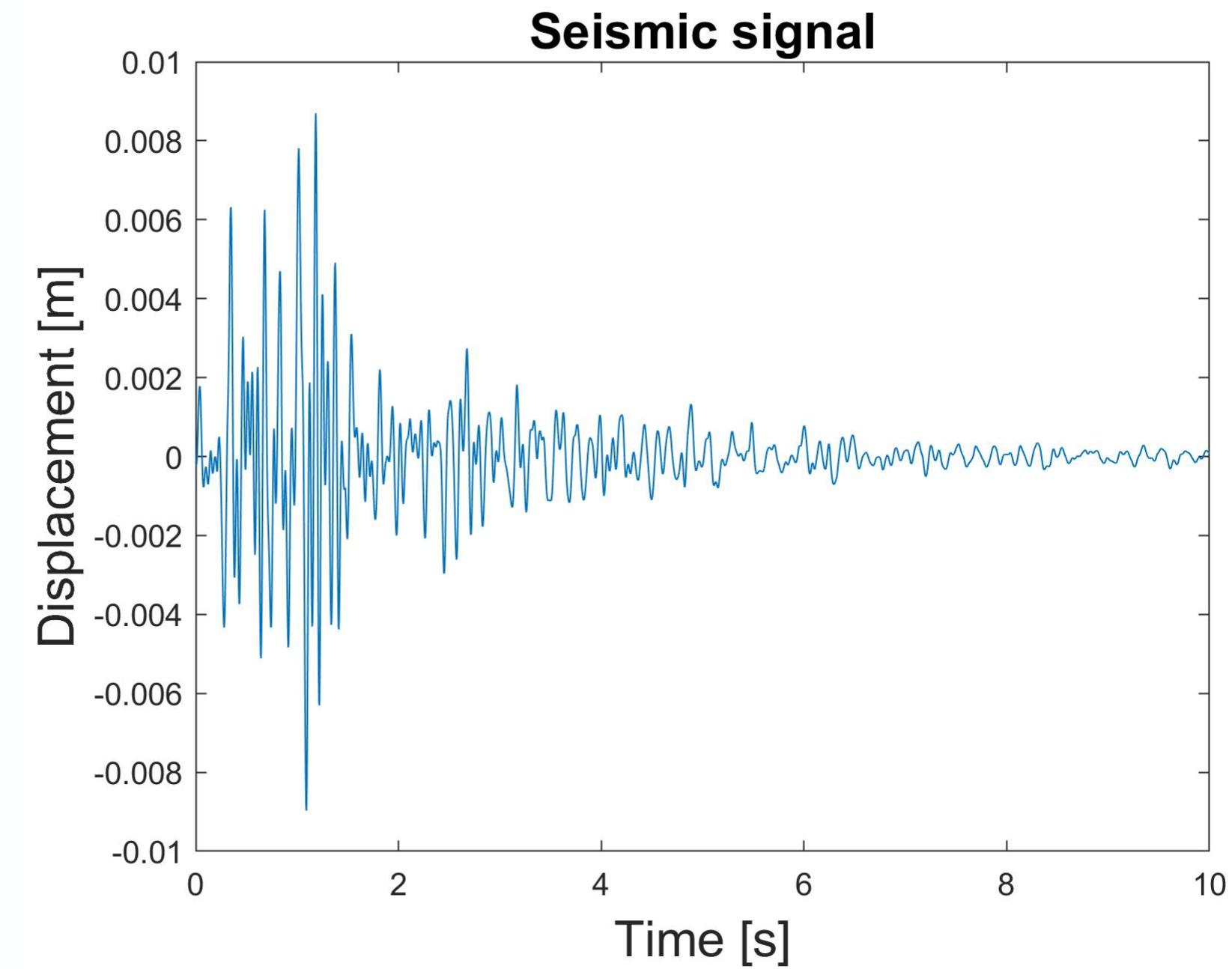


Reinforced concrete structure

Seismic excitation: Imposed displacement at only one side.

Seismic illustration

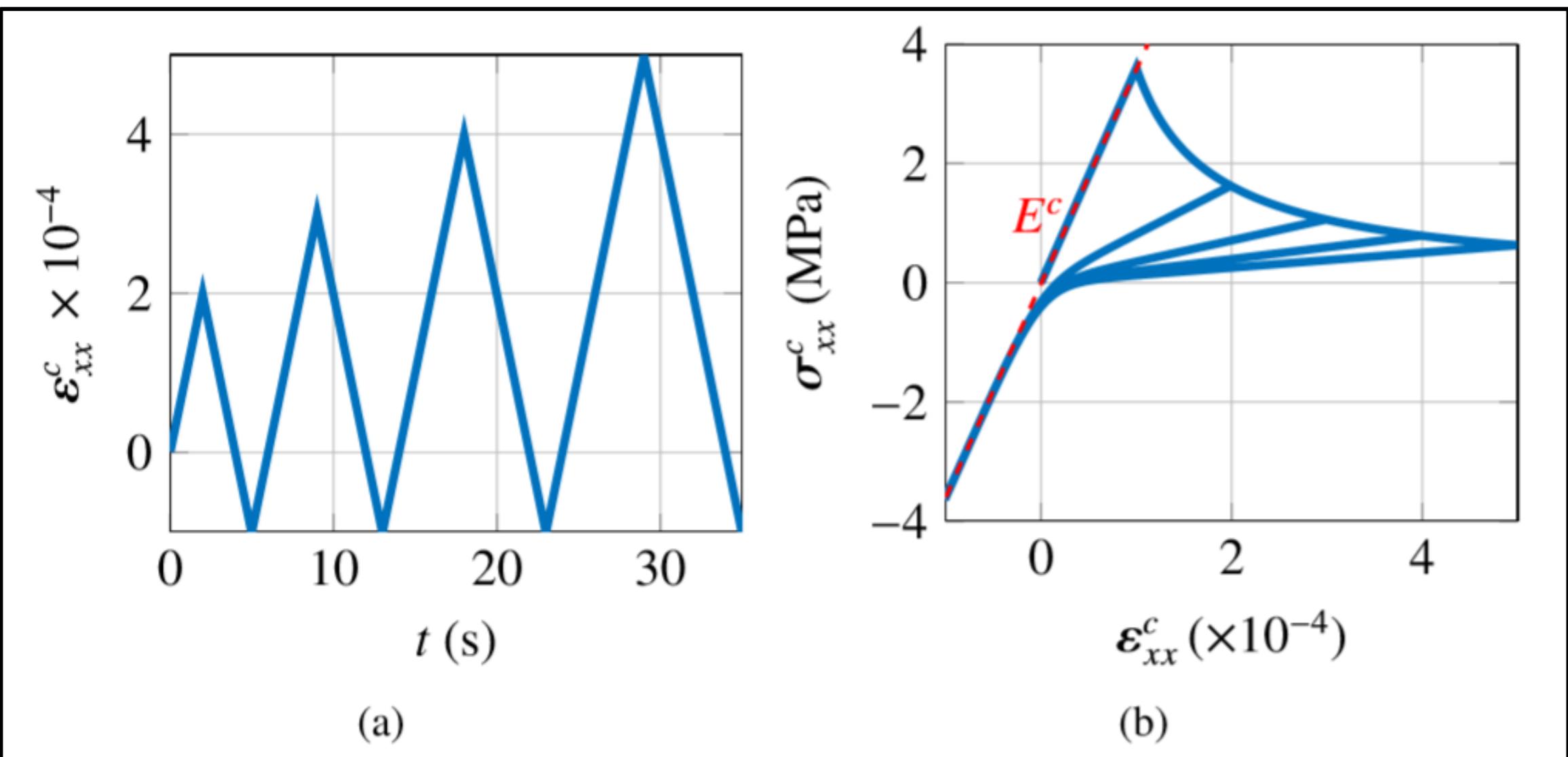
$$u_D =$$



Approximation with 7 modes (H)

Illustration

Unilateral damage model (Vassaux et al EFM 2015)



Seismic illustration

Research FE software (Rodriguez-Neron 2018)

Space dofs : 3 087

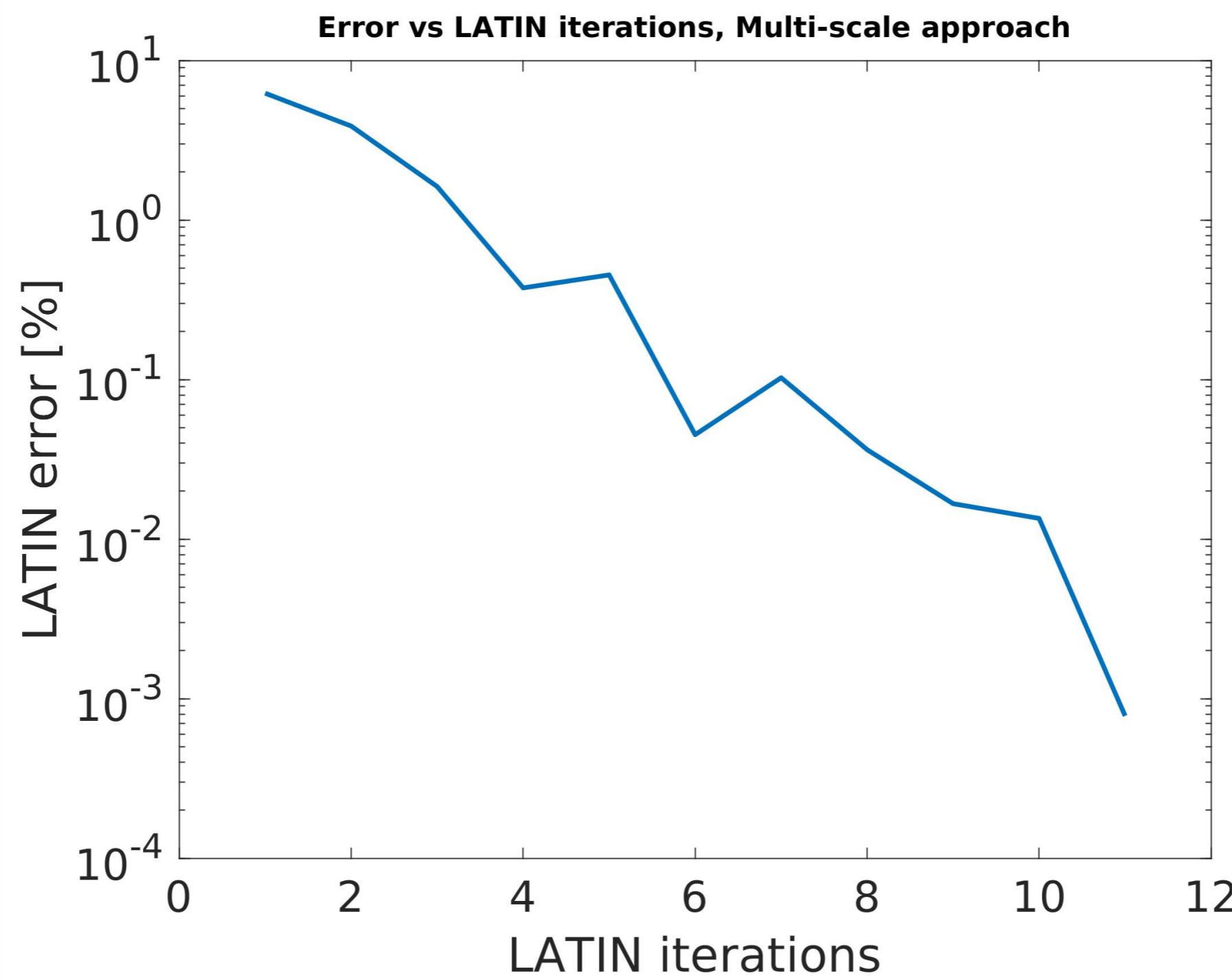
Space integration points : 34 792

Classical time discretisation : **15 000**

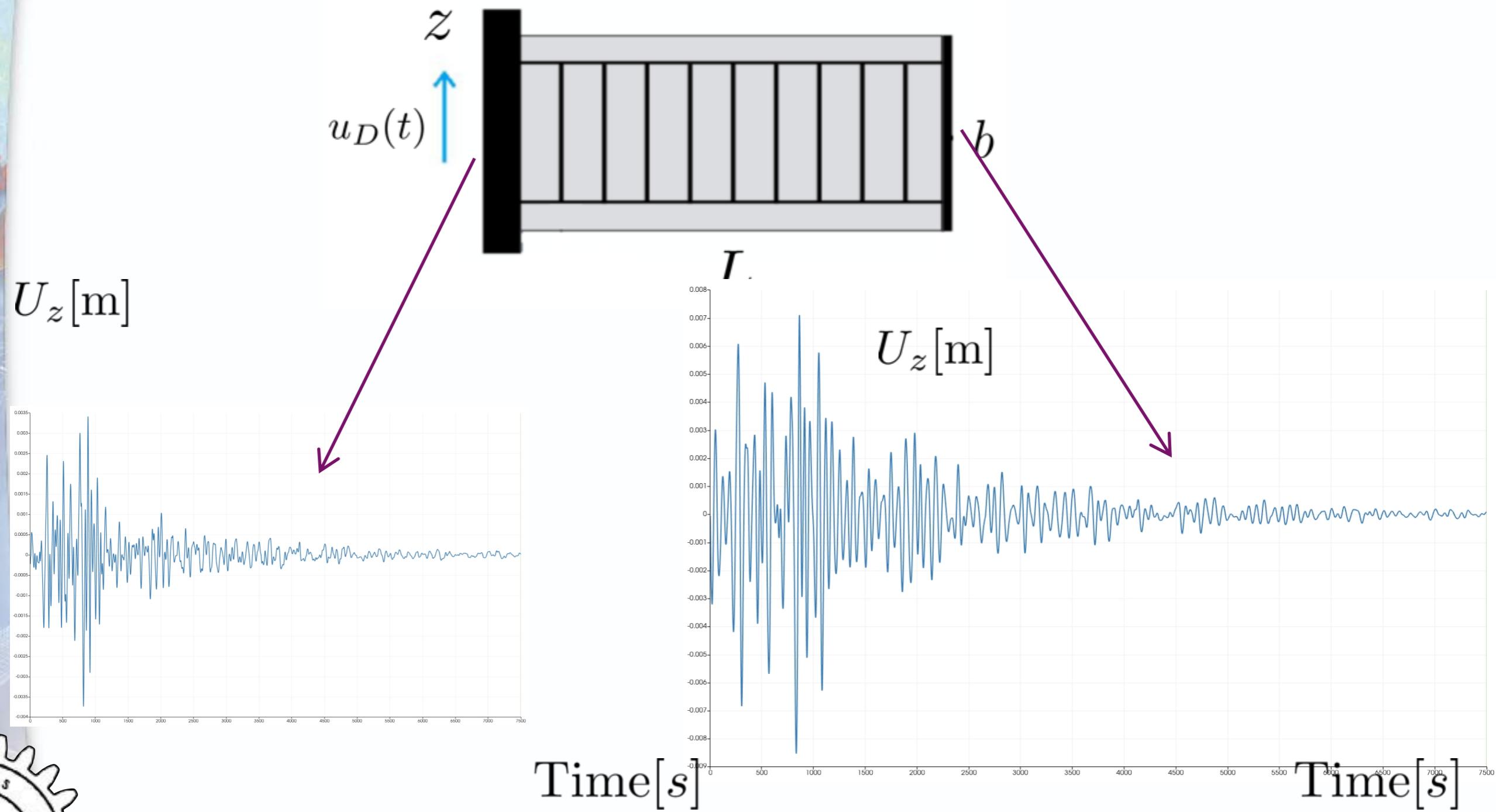
dofs with the signal theory : **885**



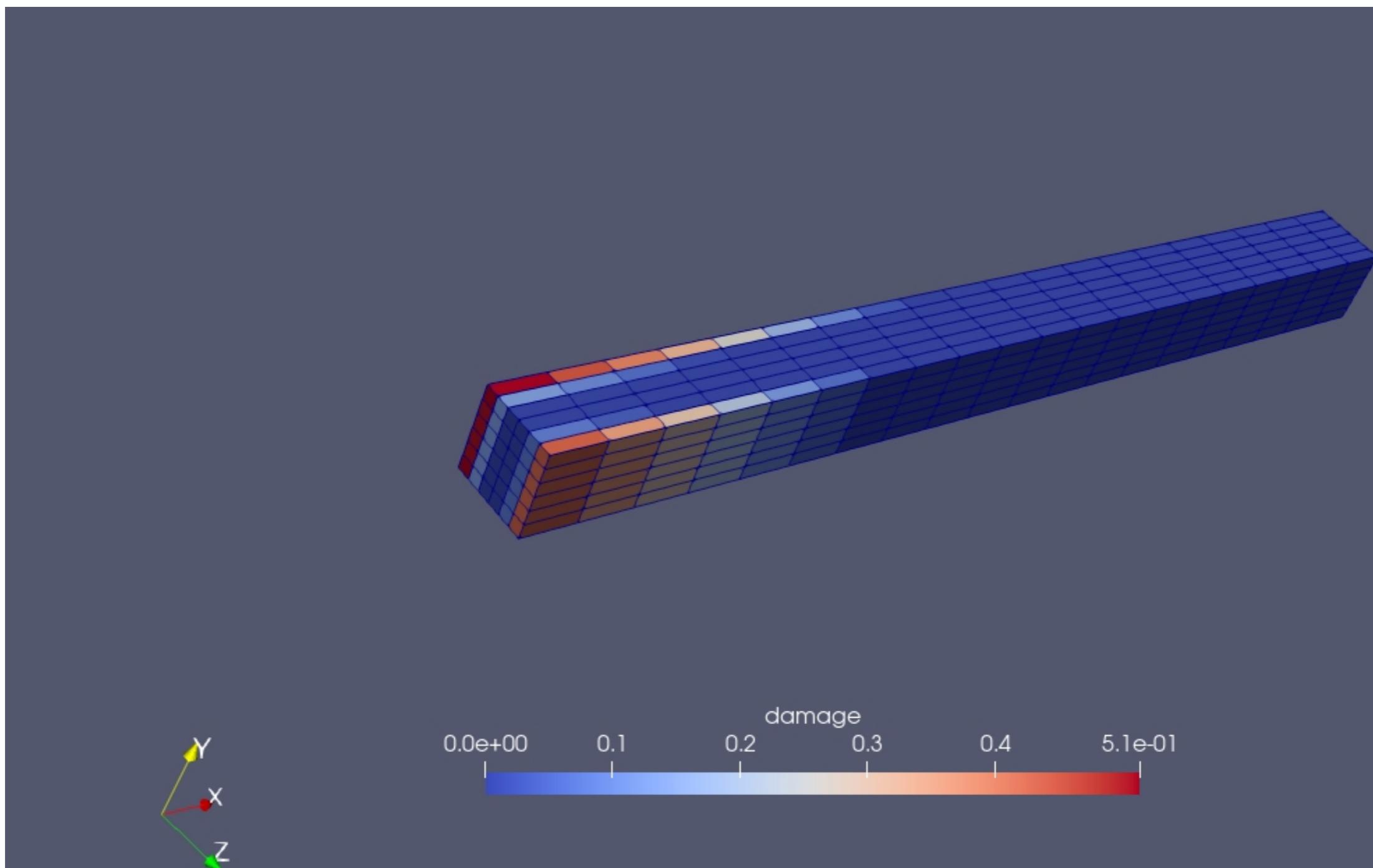
Seismic illustration



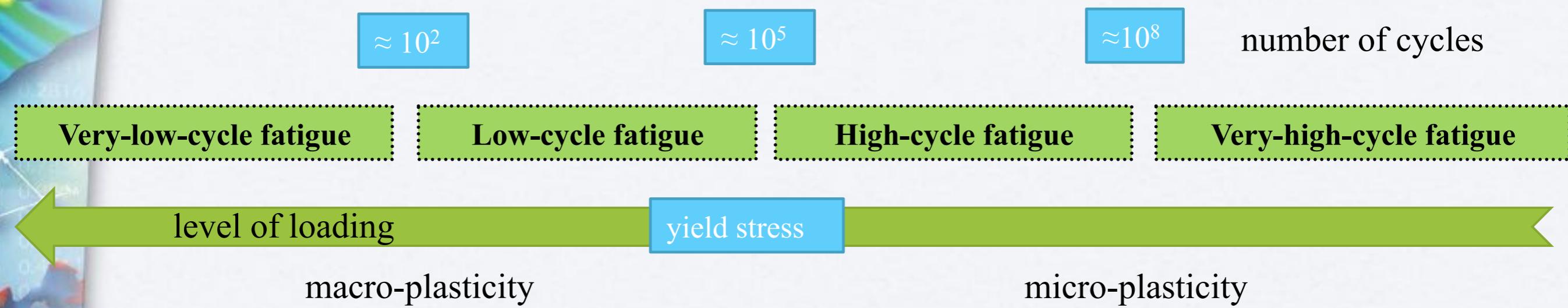
Seismic illustration



Seismic illustration



Fatigue illustrations



Fatigue computation (quasi-periodic loading) : vast literature

$$E_d(t) = \sum_{i=0}^m n(\tau, \underline{\tau}_i) S(t) A_{|\tau=t}^i \quad 1 \text{ period}$$

Fatigue illustrations

■ Cycle jumping method

—Cailletaud 1986, Chaboche 1986, Lesne-Savalle 1989, Hayburst 1994, Lemaitre-Doghri 1994, Van Paepengem et al 2001, Karlsson 2006 , Burlon et al 2014, Saanouni 2015,...

■ Time- homogenization

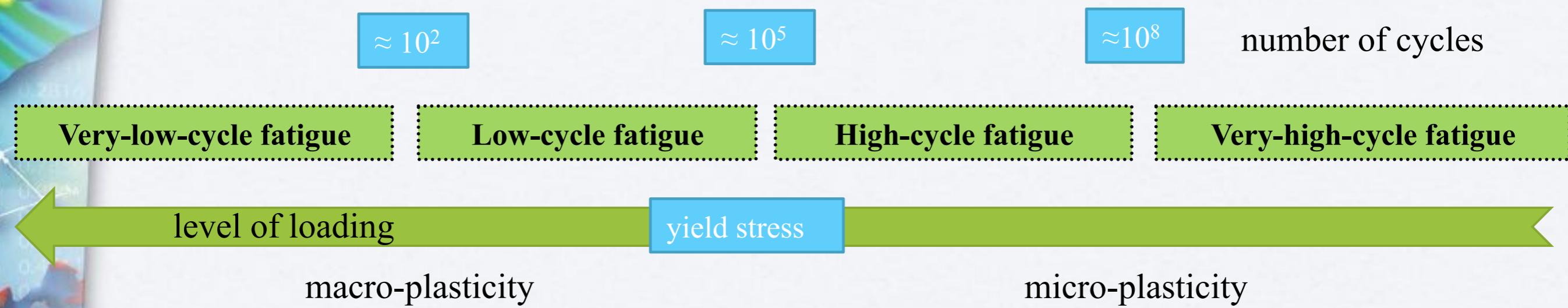
—Guennouni - Aubry 1986, Oskay-Fish 2004, Devulder et al 2010, Haoula-Doghri 2015,...

■ Time -multiscale LATIN-PGD (periodic loadings)

—Cognard-Ladeveze IJP 1993, Ladeveze 1996, Cognard et al AISI999, Maitournan et al CRAS 2002, Comte et al CRAS 2006 , Ammar et al 2017 , Bhattacharyya et al CM 2018 , CMAME 2018, Alameddin et al EJM 2019, ...



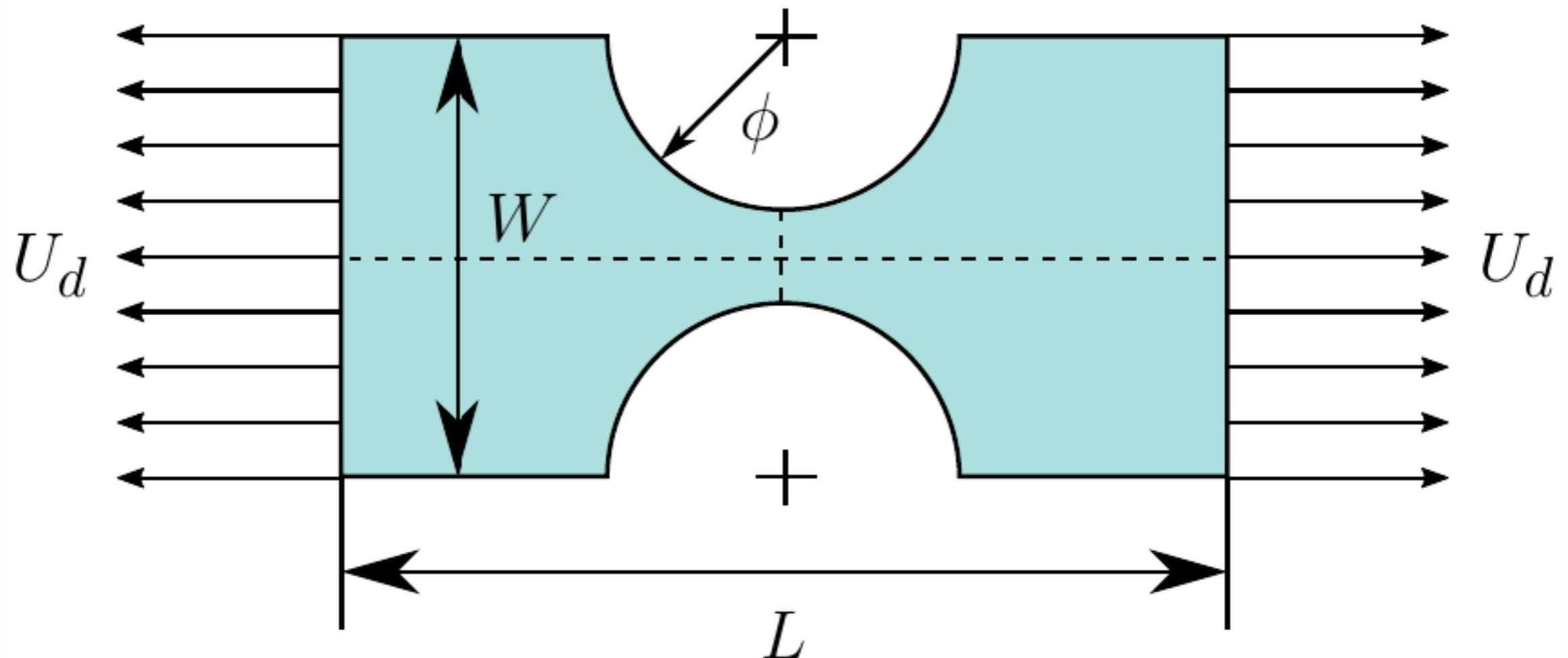
Fatigue illustrations



◆ Numerical test-Case1 : low cycle fatigue

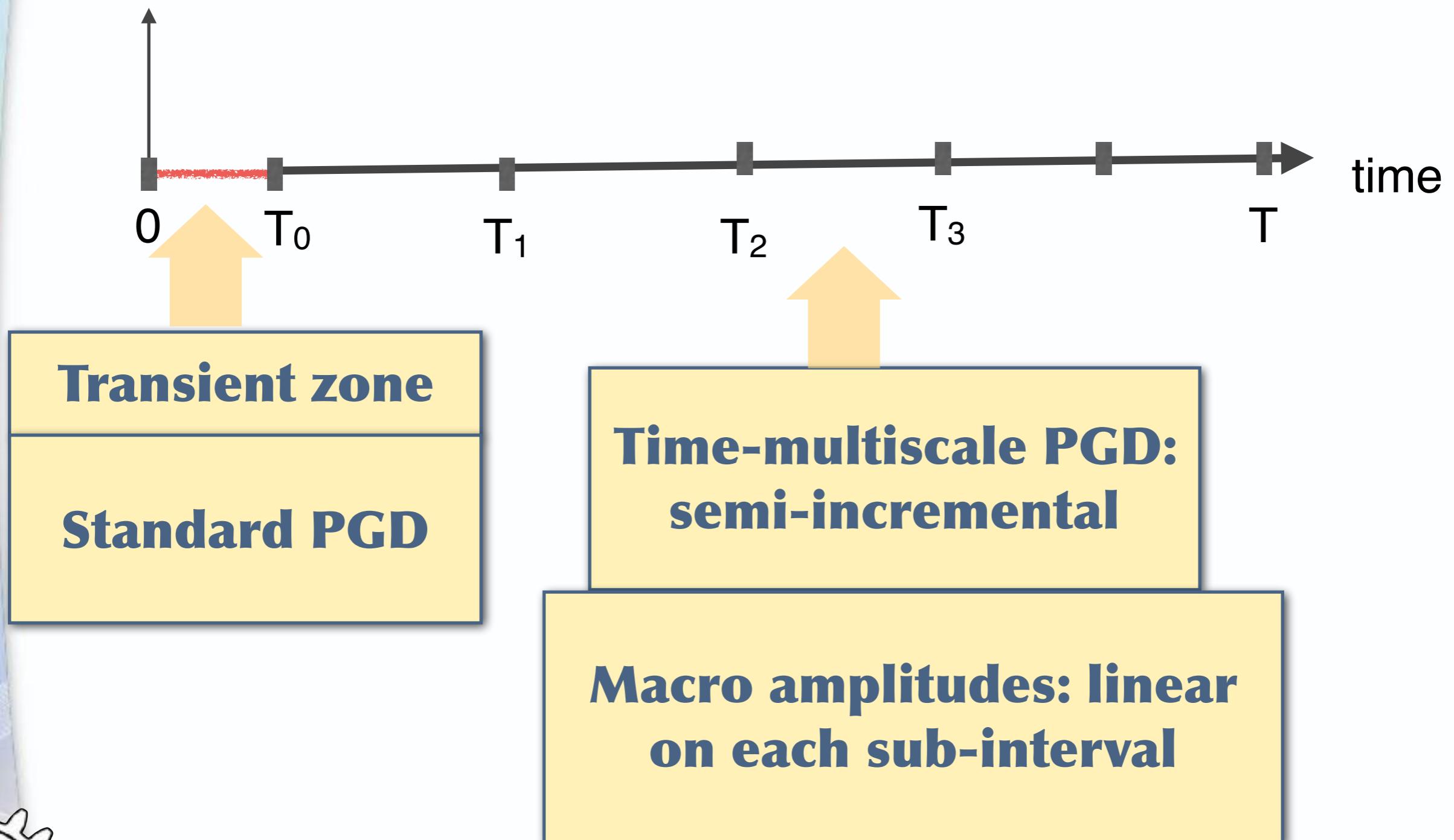
Academic example : 3D -plate with hole

Alameddin et al 2018



Viscoplasticity(Chaboche law) + unilateral fatigue damage (Lemaitre)

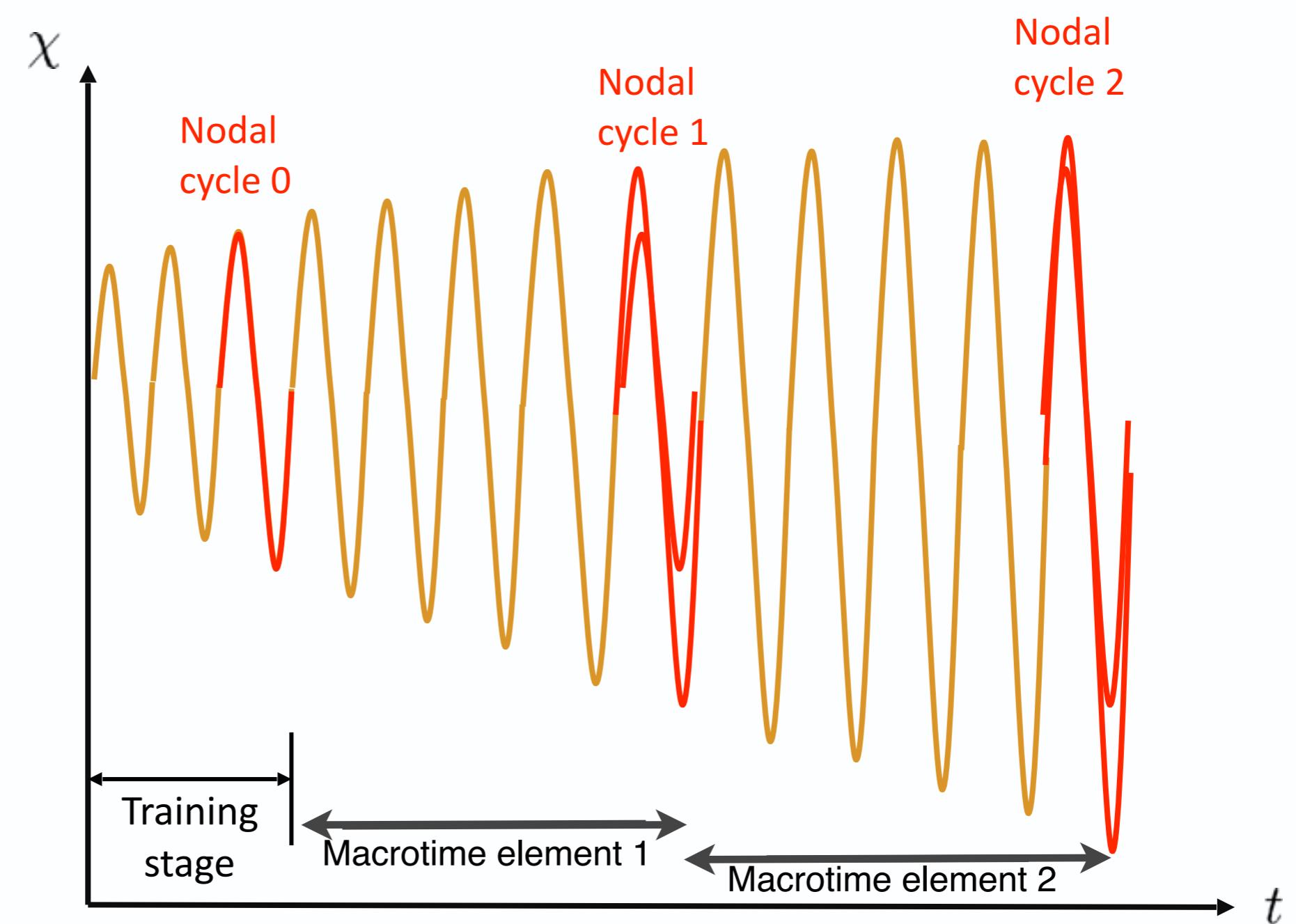
Time-multiscale PGD construction at iteration $n + 1$



Time-multiscale PGD construction at iteration $n + 1$

Computation scenario

The loading (fatigue)



Low cycle fatigue

number of cycles :738 715

semi-incremental approach (86 time elements)

macrotime : piecewise linear

accuracy : 10^{-5}

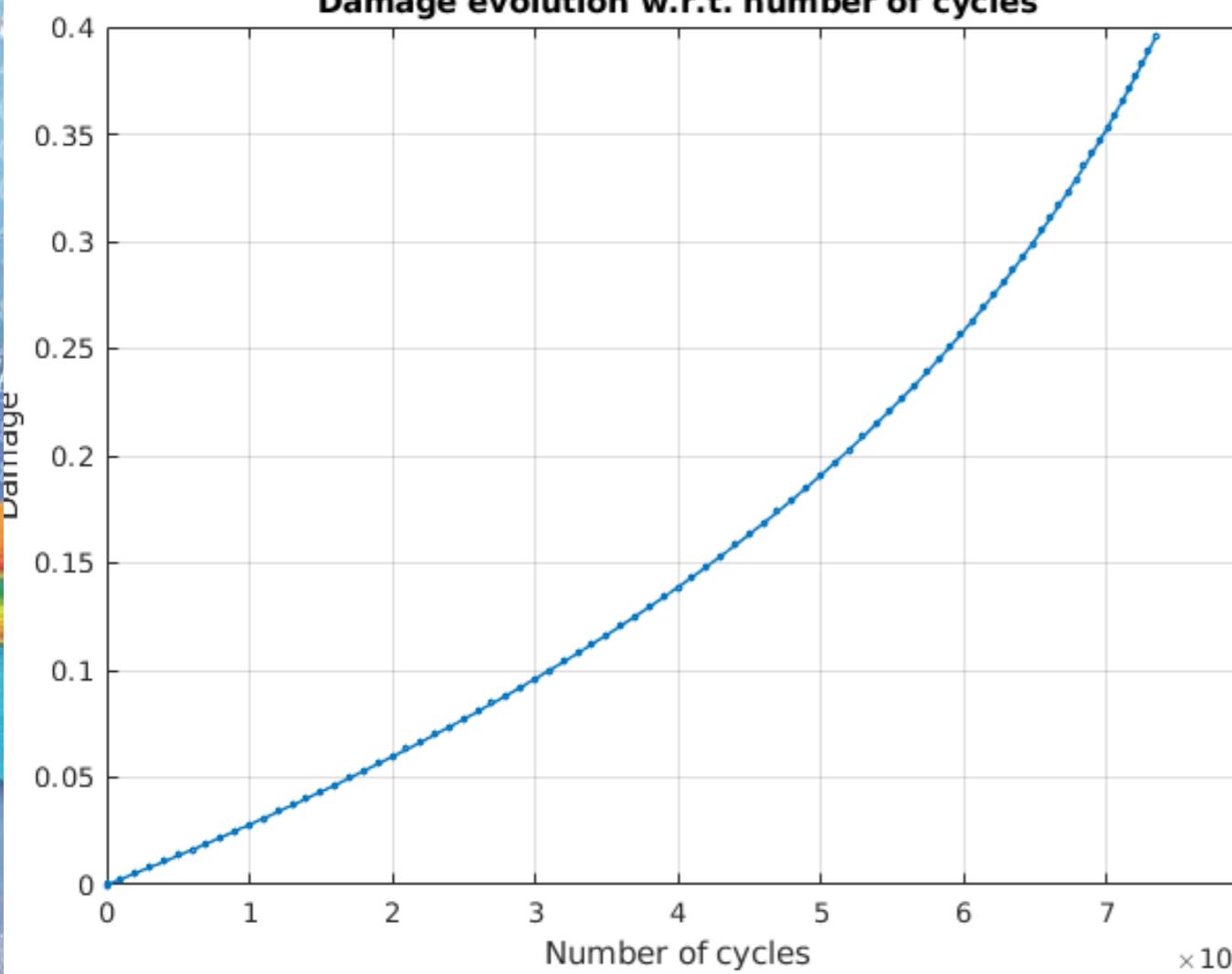
Low cycle fatigue

number of computed cycles :86 (738 715)

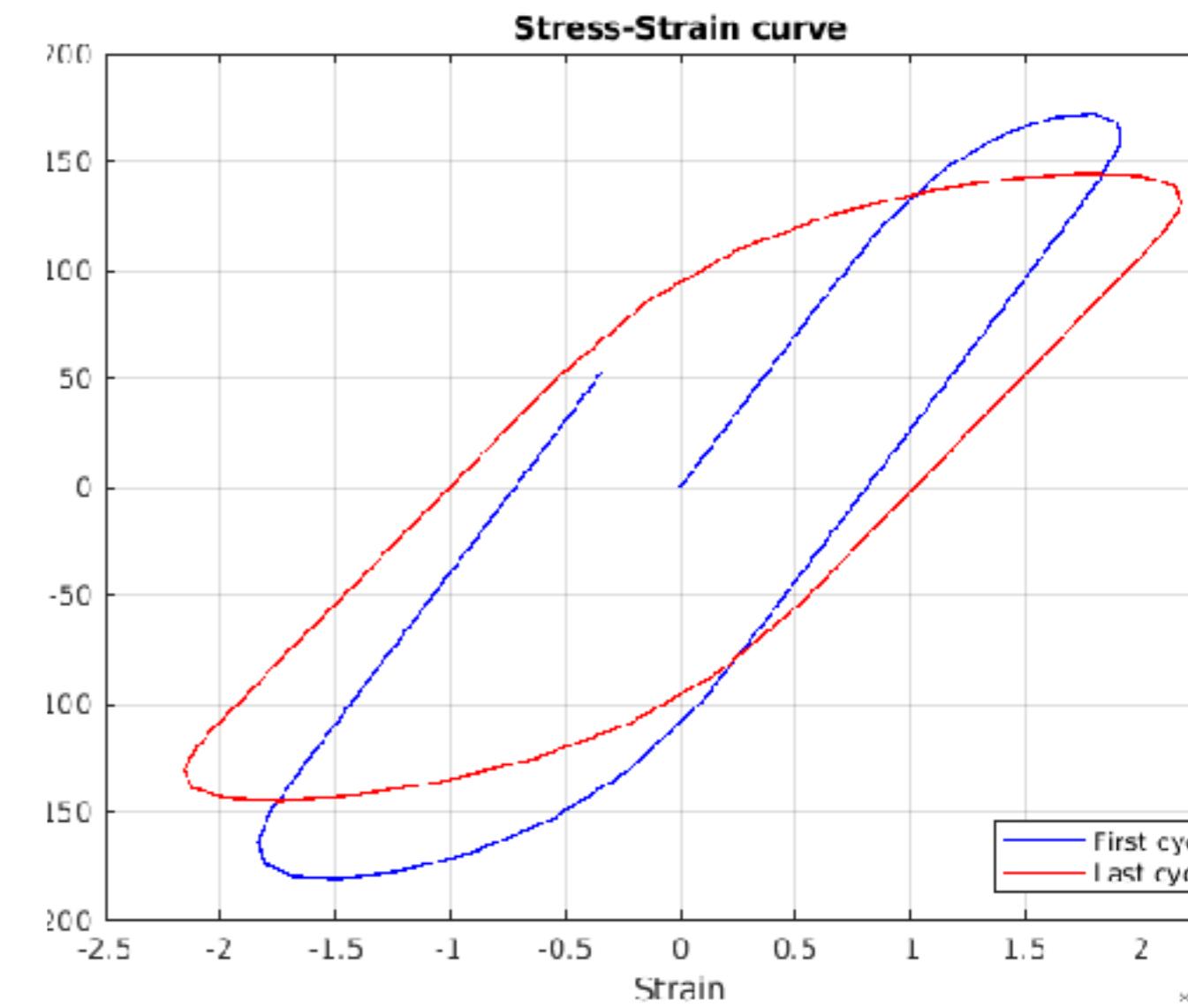
number of PGD modes / per cycle : much less than 10

Low cycle fatigue

Damage evolution w.r.t. number of cycles



Stress-Strain curve



Outline

1. A signal theory :a ROM for the loading

2. A new PGD approach : multiscale in time and non intrusive

3. First illustrations

4. Conclusion-Prospects

Conclusion-Prospects

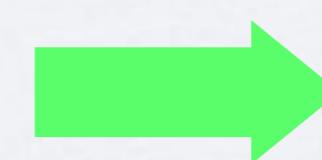
Associated challenging questions

- family of loadings with parameters (random)

$$F_d(t) = \sum_{i=0}^m n(\tau, \tau_i) S(t) A_{|\tau=t}^i \times Z^i$$



Complexity reduction ?



Engineering virtual charts ?



The end
thank you

